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Characteristic Function



Given a [subset](#) A of a larger set, the characteristic function χ_A is defined to be identically one on A . Characteristic functions are sometimes denoted using the so-called [Iverson bracket](#), and can be useful as easier to say, for example, "the characteristic function of the primes" rather than repeating a given function is a special case of a [simple function](#).

The term characteristic function is used in a different way in probability, where it is denoted $\phi(t)$ and is the [transform](#) of the [probability density function](#) using [Fourier transform](#) parameters $(a, b) = (1, 1)$,

$$\begin{aligned} \phi(t) &= \mathcal{F}_x[P(x)](t) = \int_{-\infty}^{\infty} e^{itx} P(x) dx \\ &= \int_{-\infty}^{\infty} P(x) dx + it \int_{-\infty}^{\infty} x P(x) dx + \frac{1}{2} (it)^2 \int_{-\infty}^{\infty} x^2 P(x) dx + \dots \\ &= \sum_{k=0}^{\infty} \frac{(it)^k}{k!} \mu'_k \\ &= 1 + it \mu'_1 - \frac{1}{2} t^2 \mu'_2 - \frac{1}{3!} i t^3 \mu'_3 + \frac{1}{4!} t^4 \mu'_4 + \dots \end{aligned}$$

where μ'_n (sometimes also denoted ν_n) is the n th [moment](#) about 0 and $\mu'_0 \equiv 1$ (Abramowitz and Stegun 1995). A [statistical distribution](#) is *not* uniquely specified by its [moments](#), but is uniquely specified by

$$P(x) = \mathcal{F}_t^{-1}[\phi(t)](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \phi(t) dt$$

(Papoulis 1984, p. 155).

The characteristic function can therefore be used to generate [raw moments](#),

$$\phi^{(n)}(0) \equiv \left[\frac{d^n \phi}{dt^n} \right]_{t=0} = i^n \mu'_n$$

or the [cumulants](#) κ_n ,

$$\ln \phi(t) \equiv \sum_{n=0}^{\infty} \kappa_n \frac{(it)^n}{n!}.$$

SEE ALSO: [Cumulant](#), [Iverson Bracket](#), [Moment](#), [Moment-Generating Function](#), [Probability Density Function](#) [Pages Linking Here]

Portions of this entry contributed by [Todd Rowland](#)

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CITE THIS AS:

Eric W. Weisstein et al. "Characteristic Function." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com>