Neurčitý integrál - základní vzorce

1. $\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C, \quad x > 0, n \in \mathbf{R}, \mathrm{n} \neq -1.$
2. $\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + C, \quad x \neq 0.$
3. $\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C.$
4. $\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C, \quad a > 0, a \neq 1.$
5. $\int \sin x \,\mathrm{d}x = -\cos x + C.$
6. $\int \cos x \,\mathrm{d}x = \sin x + C.$
7. $\int \frac{1}{\sin^2 x} \,\mathrm{d}x = -\mathrm{cotg}\, x + C, \quad x \neq k\pi, k \in \mathbf{Z}.$
8. $\int \frac{1}{\cos^2 x} \,\mathrm{d}x = \mathrm{tg}\, x + C, \quad x \neq \frac{\pi}{2} + k\pi, k \in \mathbf{Z}.$
9. $\int \frac{1}{1+x^2} \,\mathrm{d}x = \mathrm{arctg}\, x + C_1 = -\mathrm{arccotg}\, x + C_2.$
10. $\int \frac{1}{\sqrt{1-x^2}} \,\mathrm{d}x = \arcsin x + C_1 = -\arccos x + C_2, \quad x \in (-1, 1).$
11. $\int \frac{1}{\sqrt{x^2+a}} \,\mathrm{d}x = \ln\left(x + \sqrt{x^2+a}\right) + C, \quad a > 0.$
12. $\int \frac{1}{x^2+a^2} \,\mathrm{d}x = \frac{1}{a}\mathrm{arctg}\, \frac{x}{a} + C, \quad a \neq 0.$
13. $\int \frac{f'(x)}{f(x)} \,\mathrm{d}x = \ln|f(x)|.$

Přímá integrace
- časté využití základního vzorce č. 13, tj. $\int \frac{f'(x)}{f(x)} \,\mathrm{d}x = \ln|f(x)| + C$

1. $\int \frac{5x^2-3}{\sqrt{x}} \,\mathrm{d}x.$
2. $\int \left(3x^2 - 2x + \frac{1}{\sqrt{x^3}}\right) \,\mathrm{d}x.$
3. $\int \frac{3^x \cos^2 x - 5}{\cos^2 x} \,\mathrm{d}x.$
4. $\int \mathrm{cotg}\, x \,\mathrm{d}x.$
5. $\int \frac{\sqrt{x^4+2+x^{-4}}}{x^3} \,\mathrm{d}x.$
6. $\int \frac{1}{x \ln x} \,\mathrm{d}x.$
7. $\int x(x-2)(x-3) \,\mathrm{d}x.$

Integrace pomocí substituce

- často nutno využít vzorce $\sin^2 x = \frac{1-\cos 2x}{2}, \cos^2 x = \frac{1+\cos 2x}{2}$

8. $\int \frac{\sin^2 x}{\cos^4 x}\,\mathrm{d}x$; $\left[\ \frac{1}{3}\mathrm{tg}^{\,3}x,\ \textit{(substituce: } \mathrm{tg}\ x = t\textit{)}\right]$
9. $\int \frac{\sqrt[3]{\mathrm{arctg}\ x}}{1+x^2}\,\mathrm{d}x$; $\left[\ \frac{3}{4}\sqrt[3]{(\mathrm{arctg}\ x)^4},\ \textit{(substituce: } \mathrm{arctg}\ x = t\textit{)}\right]$
10. $\int \frac{1}{\sqrt{4x+9}}\,\mathrm{d}x$; $\left[\ \frac{1}{2}\sqrt{4x+9},\ \textit{(substituce: } 4x+9 = t\textit{)}\right]$
11. $\int \frac{1}{7x-9}\,\mathrm{d}x$; $\left[\ \frac{1}{7}\ln|7x-9|,\ \textit{(substituce: } 7x-9 = t\textit{)}\right]$
12. $\int \mathrm{e}^x \cos(\mathrm{e}^x)\,\mathrm{d}x$; $\left[\ \sin(\mathrm{e}^x),\ \textit{(substituce: } \mathrm{e}^x = t\textit{)}\right]$
13. $\int \frac{\mathrm{e}^{\frac{1}{x}}}{x^2}\,\mathrm{d}x$; $\left[\ -\mathrm{e}^{\frac{1}{x}},\ \textit{(substituce: } \frac{1}{x} = t\textit{)}\right]$
14. $\int \frac{x^3}{\sqrt{1-x^8}}\,\mathrm{d}x$; $\left[\ \frac{1}{4}\arcsin x^4,\ \textit{(substituce: } x^4 = t\textit{)}\right]$
15. $\int 2x\sqrt{x^2+1}\,\mathrm{d}x$; $\left[\ \frac{2}{3}\sqrt{(x^2+1)^3},\ \textit{(substituce: } x^2+1 = t^2\textit{)}\right]$
16. $\int \frac{1}{x\cdot\ln x\cdot\ln(\ln x)}\,\mathrm{d}x$; $\left[\ \ln|\ln(\ln x)|\right]$
17. $\int \frac{2^x\,\mathrm{d}x}{\sqrt{1+4^x}}$; $\left[\ \frac{1}{\ln 2}\ln(2^x+\sqrt{1+4^x}),\ \textit{(substituce: } 2^x = t\textit{)}\right]$

Integrace per partes

- umět odvodit vzorec $\int u'v\,\mathrm{d}x = uv - \int uv'\,\mathrm{d}x$ z derivace součinu uv

18. $\int x^2\cos x\,\mathrm{d}x$; $\left[(x^2-2)\sin x + 2x\cos x\right]$
19. $\int x^2\mathrm{arctg}\ x\,\mathrm{d}x$; $\left[\frac{x^3}{3}\mathrm{arctg}\ x - \frac{1}{6}x^2 + \frac{1}{6}\ln(x^2+1)\right]$
20. $\int \sin^2 x\,\mathrm{d}x$; $\left[\frac{1}{2}(x - \sin x\cos x)\right]$
21. $\int \frac{x}{\sin^2 x}\,\mathrm{d}x$; $\left[-x\mathrm{cotg}\ x + \ln|\sin x|\right]$
22. $\int \mathrm{e}^x\cos x\,\mathrm{d}x$; $\left[\frac{\mathrm{e}^x}{2}(\sin x + \cos x)\right]$
23. $\int \arcsin\ x\,\mathrm{d}x$; $\left[x\arcsin\ x + \sqrt{1-x^2}\right]$
24. $\int x^2\sin(2x)\,\mathrm{d}x$; $\left[-\frac{x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x)\right]$
25. $\int x^3\cos x\,\mathrm{d}x$; $\left[(x^3-6x)\sin x + (3x^2-6)\cos x\right]$
26. $\int \cos(\ln x)\,\mathrm{d}x$; $\left[\frac{x}{2}(\cos(\ln x) + \sin(\ln x))\right]$