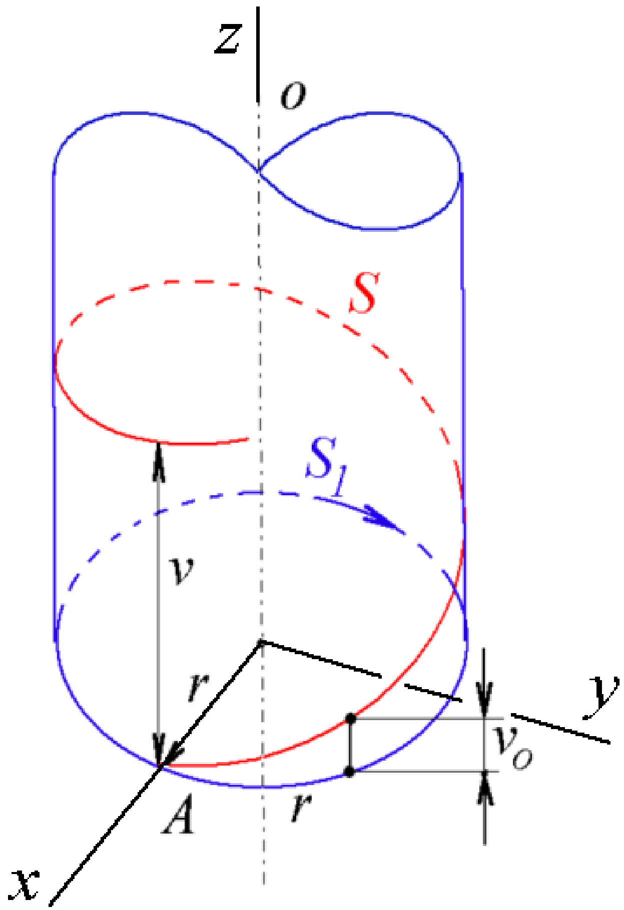
# Šroubovice

**Šroubovice** - dráha pohybu složeného z rotace bodu kolem osy a z translace ve směru této osy.




$$\boldsymbol{S}(t)=\begin{pmatrix}x\\y\\z\\1\end{pmatrix}^T=\begin{pmatrix}\cos t & -\sin t & 0 & 0\\ \sin t & \cos t & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & v_0 t\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}r\\0\\0\\1\end{pmatrix}^T=$$

$$=\begin{pmatrix}\cos t & -\sin t & 0 & 0\\ \sin t & \cos t & 0 & 0\\ 0 & 0 & 1 & v_0 t\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}r\\0\\0\\1\end{pmatrix}^T$$

$$\boldsymbol{S}(t)=\left(r\cos t; r\sin t; v_0 t; 1\right)$$

$r$ - poloměr

$v_0$ - redukovaná výška

$v_0 > 0 \Rightarrow$ pravotočivá

$v_0 < 0 \Rightarrow$ levotočivá

## Tečna šroubovice:

$$\boldsymbol{S}(t)=(r\cos t;r\sin t;v_0t;1)$$

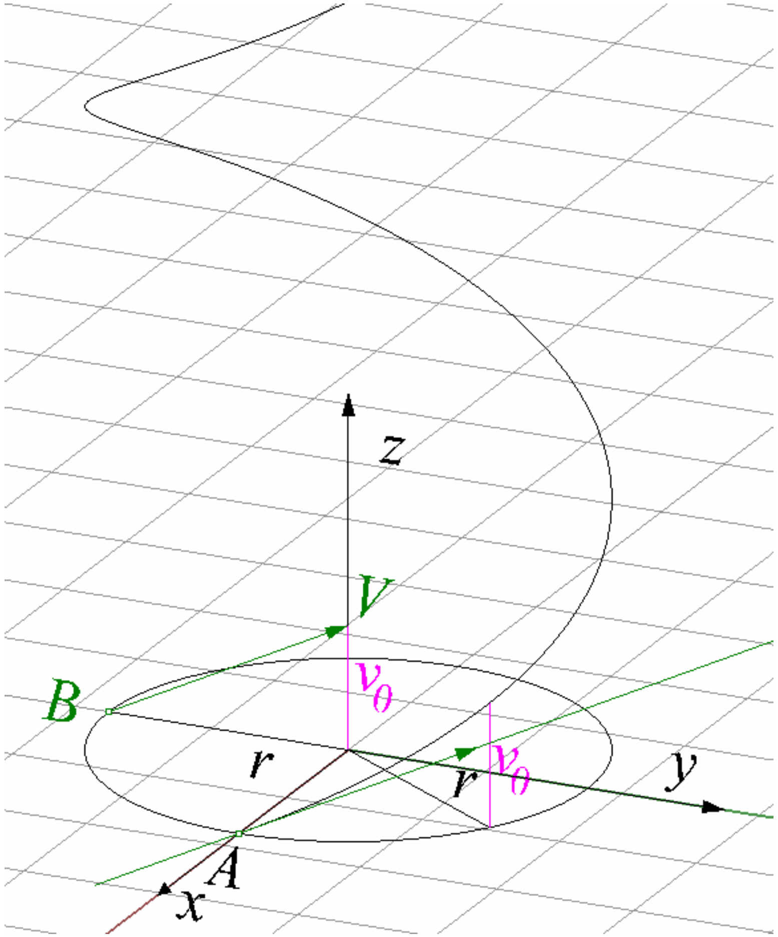
$$\boldsymbol{S}'(t)=(-r\sin t;r\cos t;v_0;0)$$

v bodě $\boldsymbol{T}_0=(r\cos t_0;r\sin t_0;v_0t_0;1)$

$$\boldsymbol{X}=\boldsymbol{T}_0+\boldsymbol{S}'(\boldsymbol{T}_0)\cdot t$$

$$\boldsymbol{X}=(r\cos t_0;r\sin t_0;v_0t_0;1)+(-r\sin t_0;r\cos t_0;v_0;0)\cdot t$$

$$(x;y;z;1)=(r\cos t_0;r\sin t_0;v_0t_0;1)+(-r\sin t_0;r\cos t_0;v_0;0)\cdot t$$




$$x=r\cos t_0-t\cdot r\sin t_0$$

$$y=r\sin t_0+t\cdot r\cos t_0$$

$$z=t\cdot v_0t_0$$

$$1=1$$

$$\boldsymbol{S}'(t)=(-r\sin t;r\cos t;v_0;0)$$

$$\boldsymbol{S}'(0)=(-r\sin 0;r\cos 0;v_0;0)$$

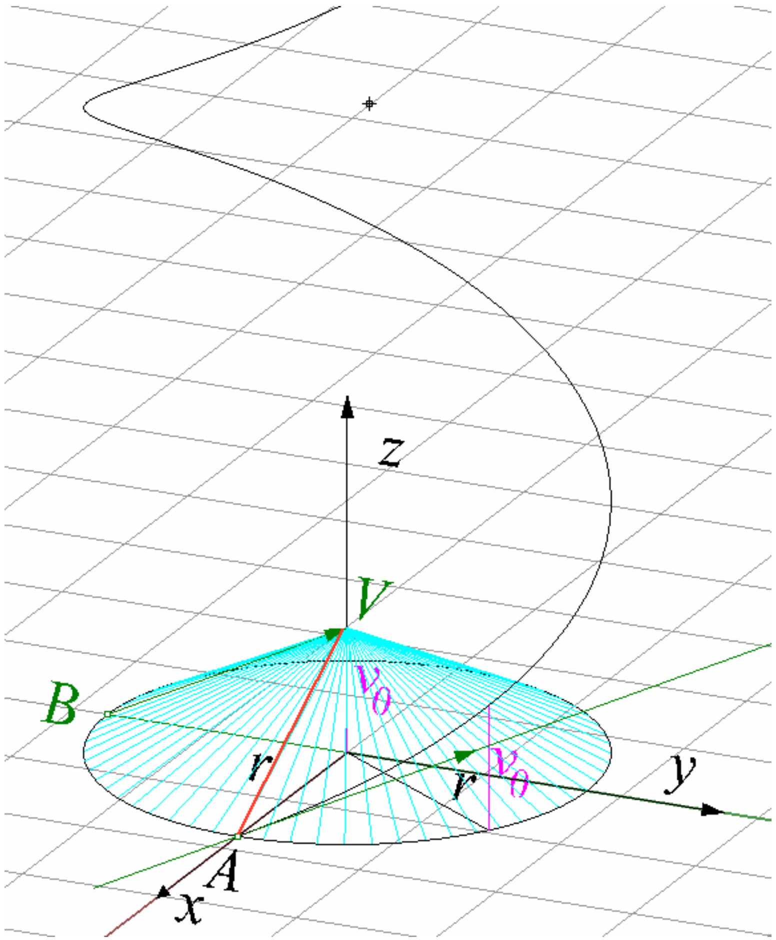
$$=(0;r;v_0;0)$$

$$\boldsymbol{S}'(0)=\boldsymbol{V}-\boldsymbol{B}$$

$$\boldsymbol{V}=(0;0;v_0;1)$$

$$\boldsymbol{B}=(0;-r;0;1)$$

## Tečna šroubovice:




$$S'(t)=(-r\sin t;r\cos t;v_0;0)$$

$$S'(0)=(0;r;v_0;0)$$

$$S'(0)=V-B$$

$$V=(0;0;v_0;1)$$

$$A=(0;-r;0;1)\ \Rightarrow\ V-A=(-r;0;v_0;0)$$

$$K(t)=\begin{pmatrix}\cos t & -\sin t & 0 & 0\\ \sin t & \cos t & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}-r\\ 0\\ v_0\\ 0\end{pmatrix}^T=\begin{pmatrix}-r\cos t\\ -r\sin t\\ v_0\\ 0\end{pmatrix}^T$$

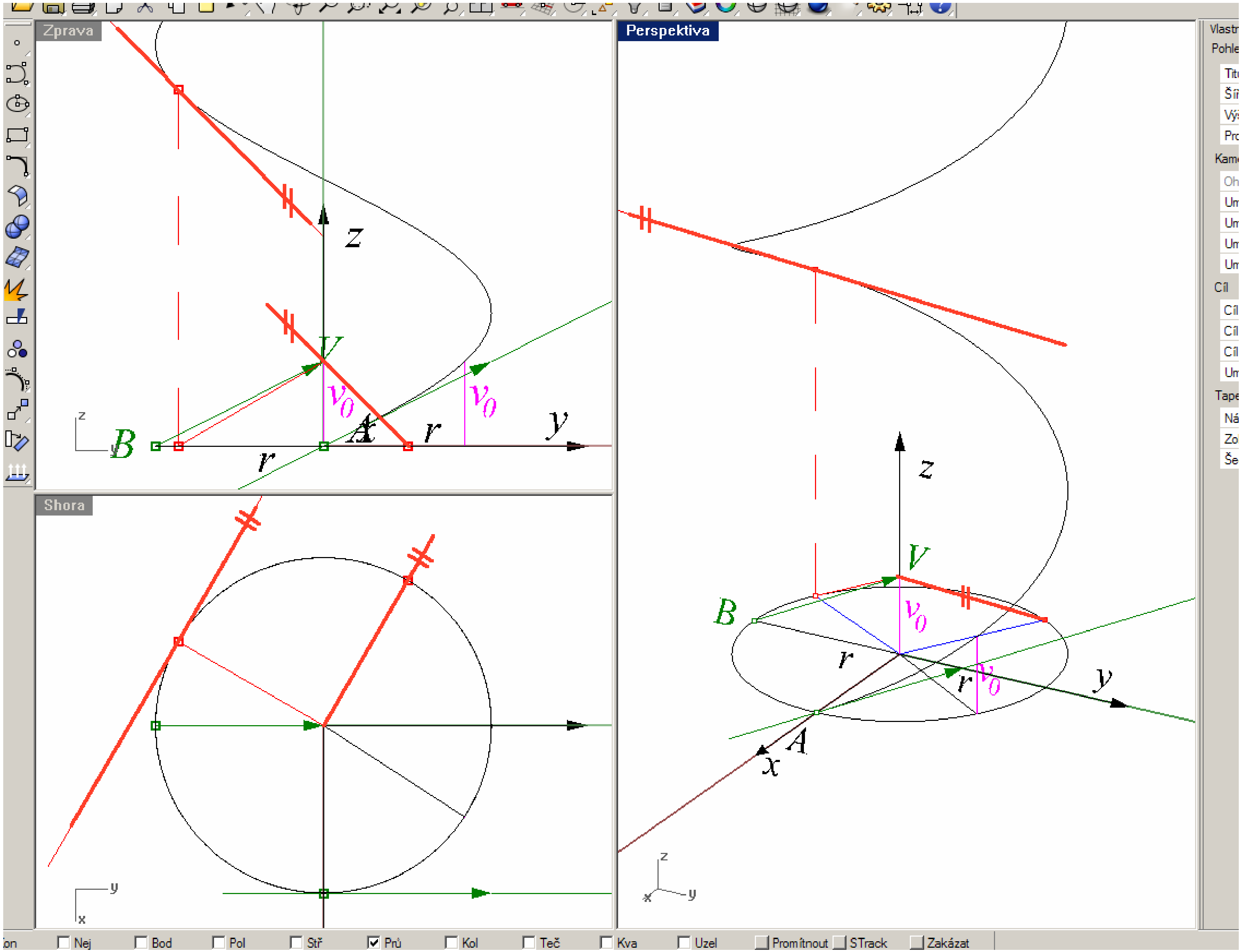
$$K(t)=(-r\cos t;-r\sin t;v_0;0)$$

$$S'(t)=(-r\sin t;r\cos t;v_0;0)$$

$$t\to t+\tfrac{\pi}{2}$$

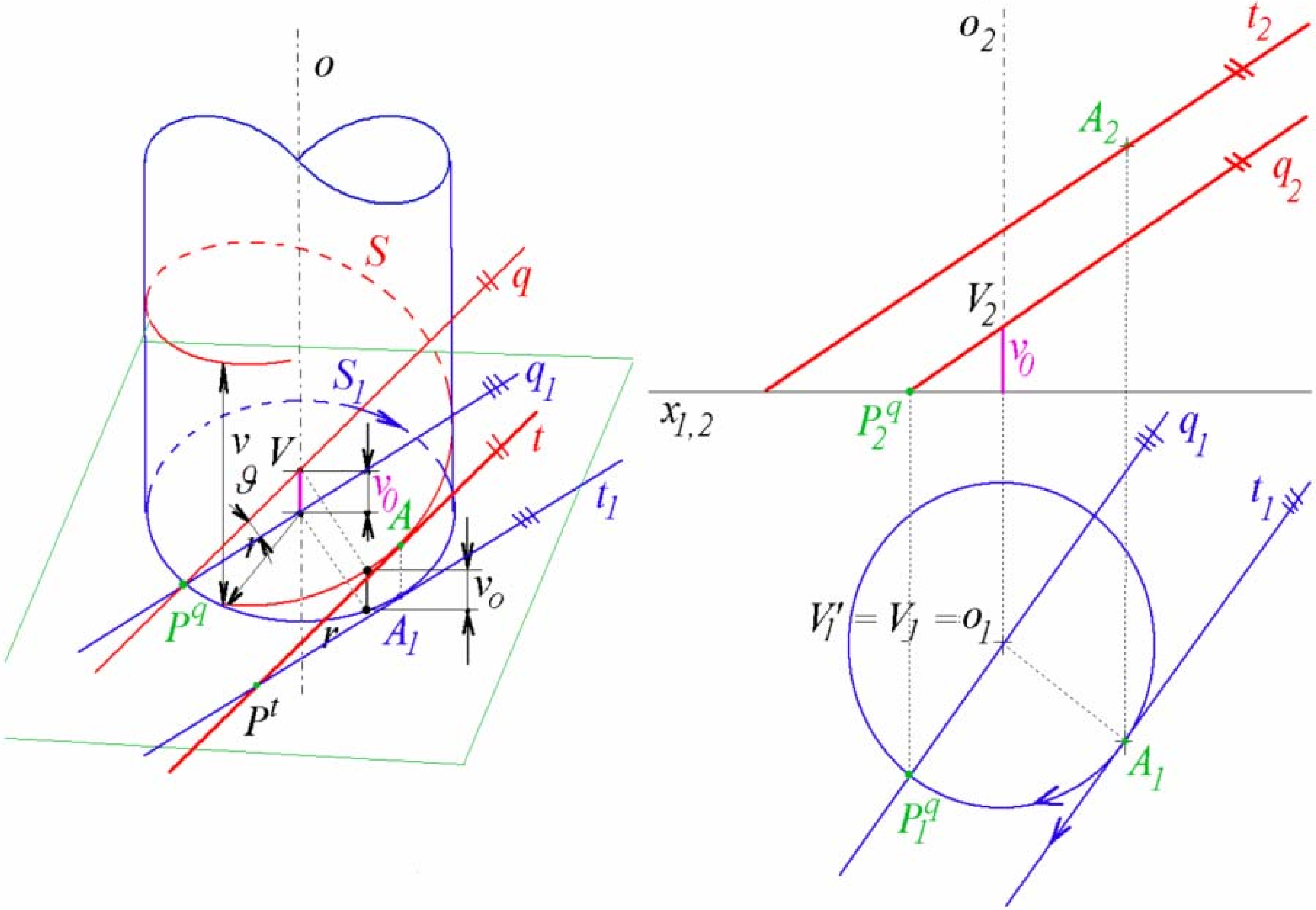
$$S'\left(t+\tfrac{\pi}{2}\right)=\left(-r\sin\left(t+\tfrac{\pi}{2}\right);r\cos\left(t+\tfrac{\pi}{2}\right);v_0;0\right)$$

$$S'\left(t+\tfrac{\pi}{2}\right)=(-r\cos t;-r\sin t;v_0;0)=K(t)$$

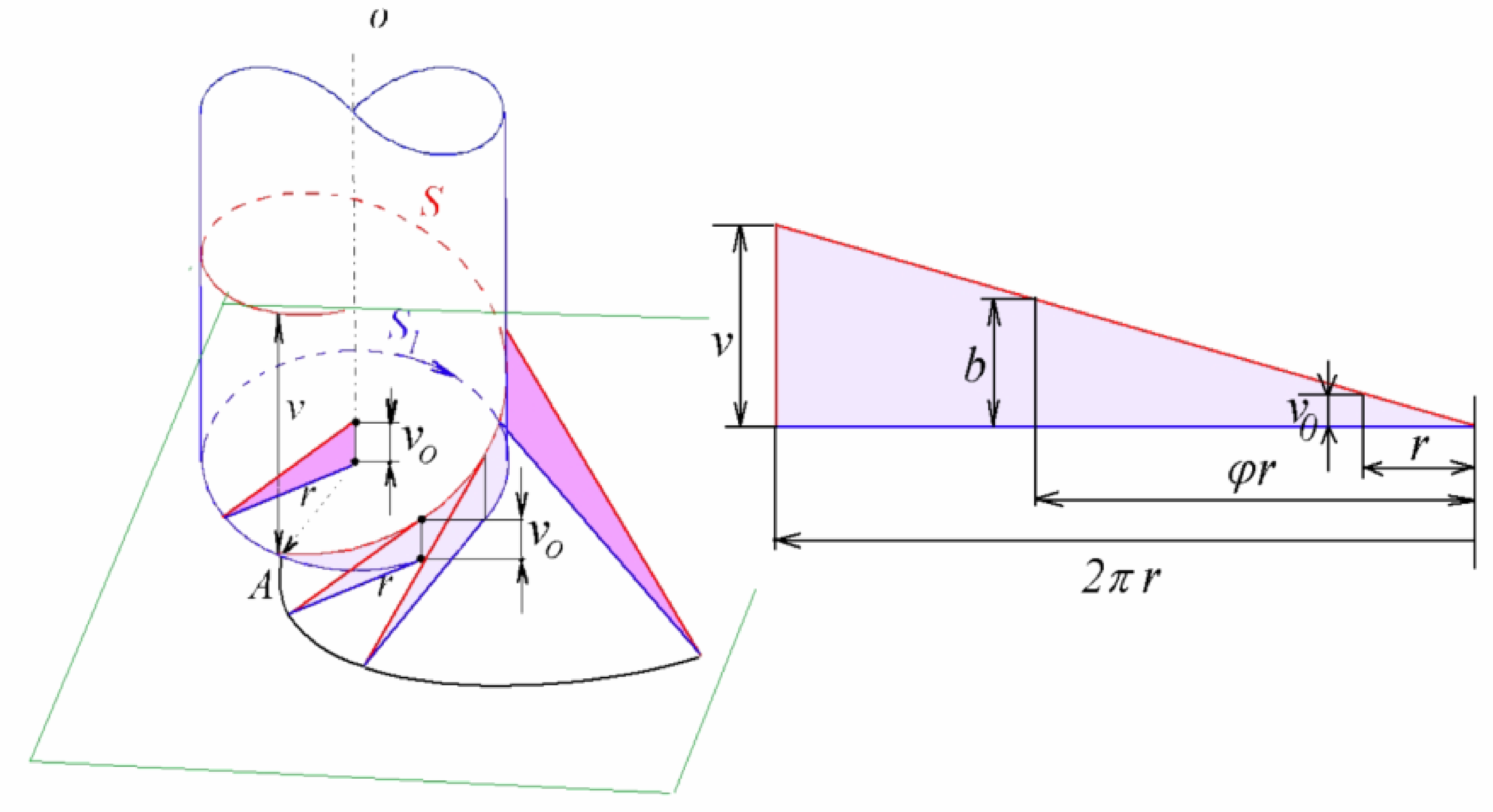


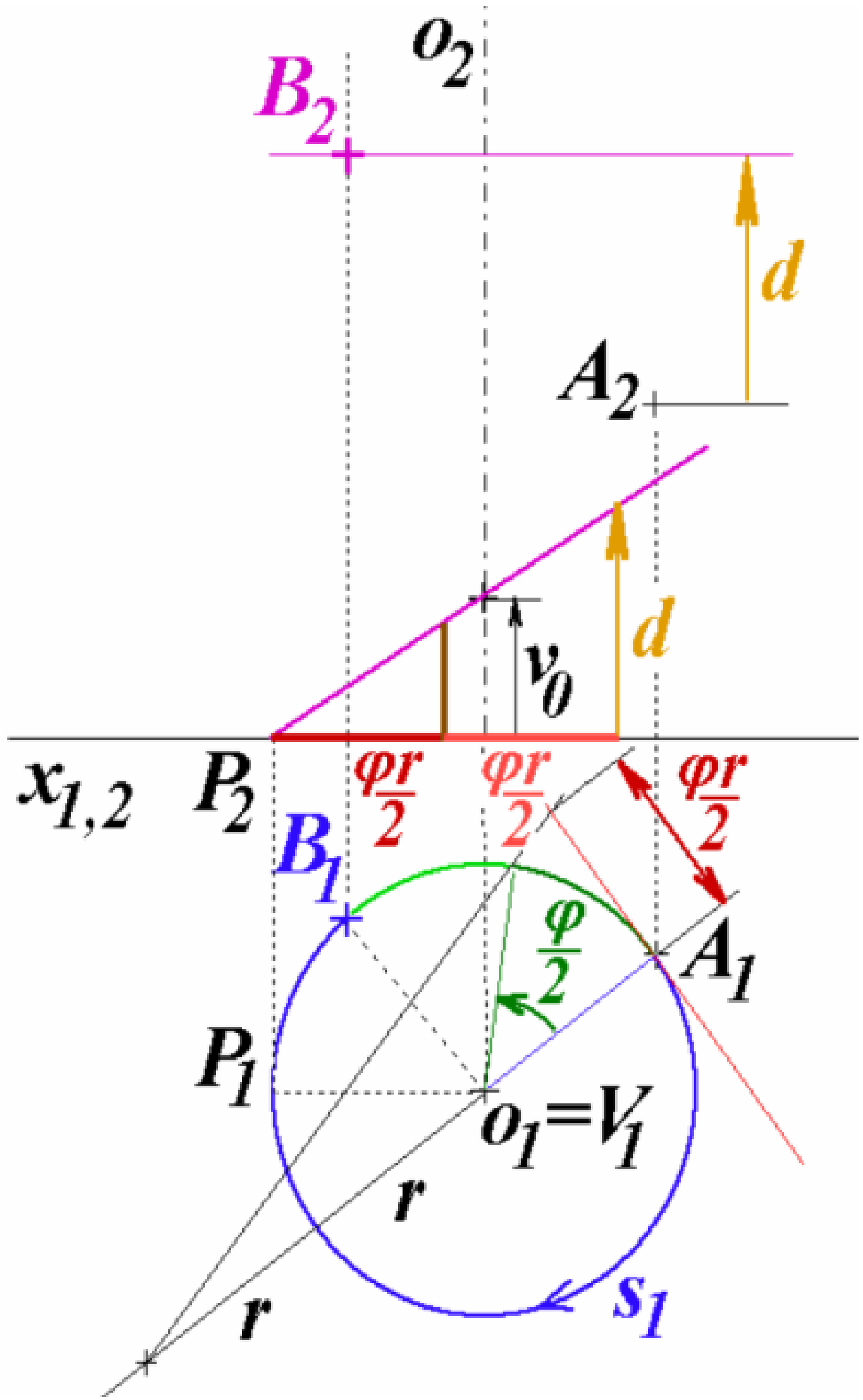
Zprava
Perspektiva
Shora
z
y
x
V
A
B
r
$v_0$
Nej
Bod
Pol
Stř
Prů
Kol
Teč
Kva
Uzel
Promítnout
STrack
Zakázat

**Tečna šroubovice v Mongeově promítání** $S(o, v_0, A, pravo)$



**Šroubování bodu:** a) o zadaný úhel
b) o zadanou výšku




o2
B2
d
A2
d
v0
x1,2
P2
φr/2
φr/2
φr/2
B1
φ/2
A1
P1
o1=V1
r
r
s1

## Oskulační rovina v bodě $T_0$:

$$\boldsymbol{X} = \boldsymbol{T}_0 + \boldsymbol{S}'(\boldsymbol{T}_0)\cdot u + \boldsymbol{S}''(\boldsymbol{T}_0)\cdot v$$

$$\boldsymbol{S}(t) = (r\cos t; r\sin t; v_0 t; 1)$$

$$\boldsymbol{S}'(t) = (-r\sin t; r\cos t; v_0; 0) \qquad \boldsymbol{S}'(\boldsymbol{T}_0) = \boldsymbol{S}'(t_0) = (-r\sin t_0; r\cos t_0; v_0; 0)$$

$$\boldsymbol{S}''(t) = (-r\cos t; -r\sin t; 0; 0) \qquad \boldsymbol{S}''(\boldsymbol{T}_0) = \boldsymbol{S}''(t_0) = (-r\cos t_0; -r\sin t_0; 0; 0)$$

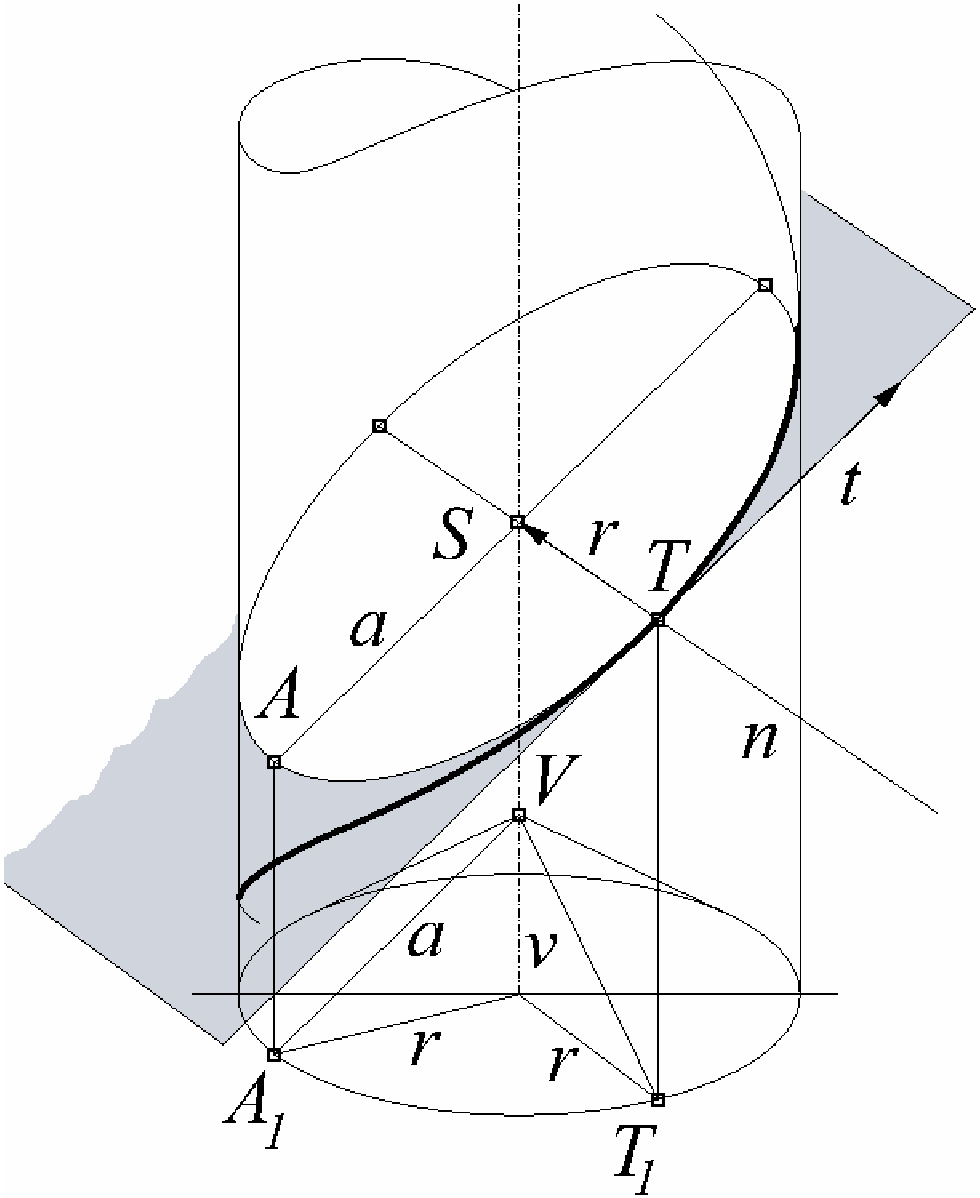
## Oskulační kružnice v bodě $T_0$:

$${}^1\kappa(t) = \frac{\left|\boldsymbol{S}'(t)\times\boldsymbol{S}''(t)\right|}{\left|\boldsymbol{S}'(t)\right|^3} = \frac{\left|(-r\sin t; r\cos t; v_0; 0)\times(-r\cos t; -r\sin t; 0; 0)\right|}{\left|(-r\sin t; r\cos t; v_0; 0)\right|^3} =$$

$$= \frac{1}{\left(\sqrt{r^2+v_0^2}\right)^3}\cdot\left\|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -r\sin t & r\cos t & v_0 \\ -r\cos t & -r\sin t & 0 \end{matrix}\right\| = \frac{1}{\left(\sqrt{r^2+v_0^2}\right)^3}\left|\mathbf{i}\cdot rv_0\sin t - \mathbf{j}\cdot rv_0\cos t + \mathbf{k}\cdot r^2\right| =$$

$$= \frac{1}{\left(\sqrt{r^2+v_0^2}\right)^3}\left|(rv_0\sin t; rv_0\cos t; r^2)\right| = \frac{\sqrt{r^2v_0^2+r^4}}{\left(\sqrt{r^2+v_0^2}\right)^3} = \frac{r\cdot\sqrt{v_0^2+r^2}}{\left(\sqrt{r^2+v_0^2}\right)^3} = \frac{r}{r^2+v_0^2}$$

$$r = \frac{1}{{}^1\kappa} = \frac{r^2+v_0^2}{r}$$



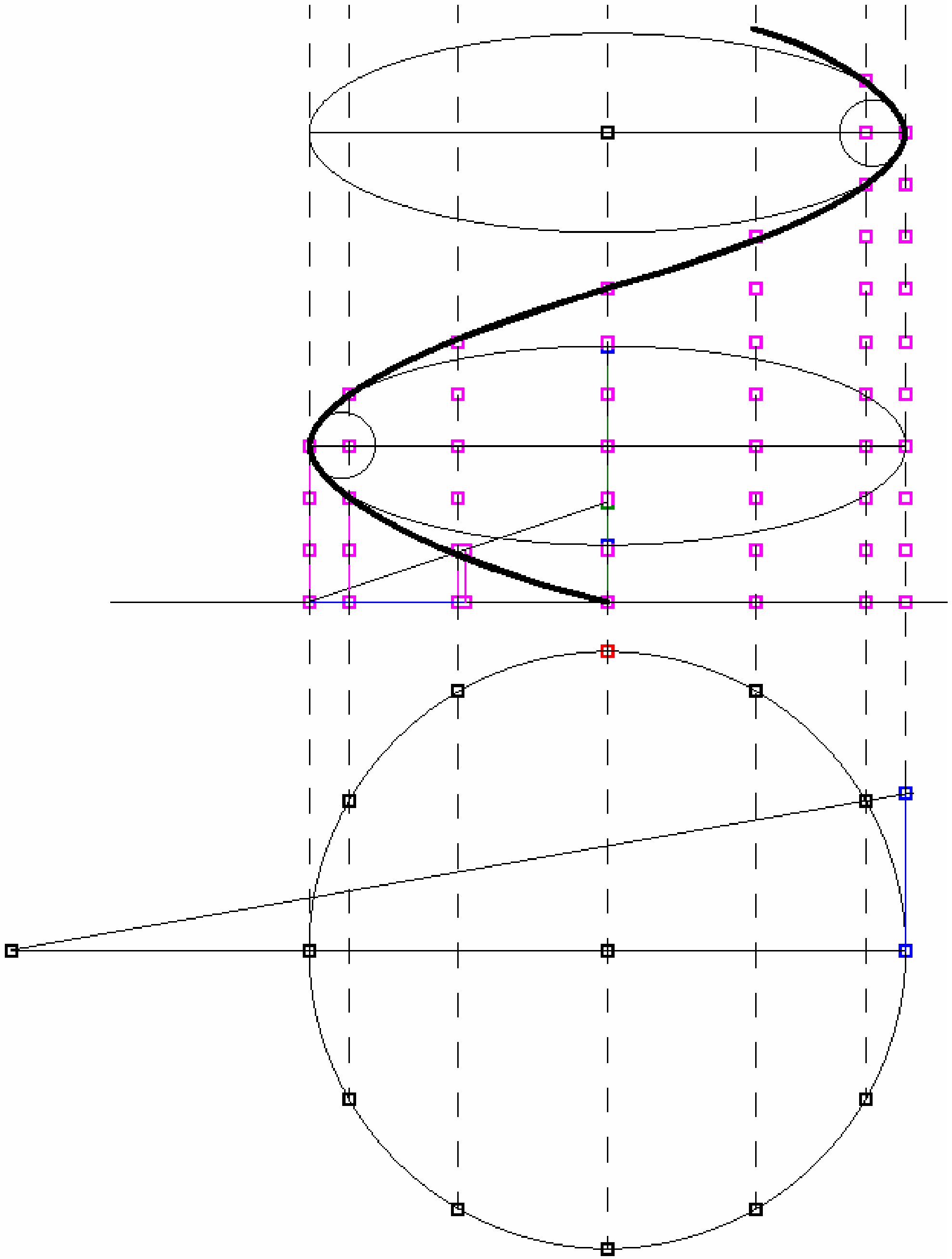

$$r = \frac{1}{{}^{1}\kappa} = \frac{r^2 + v_0^2}{r}$$

Poloměr oskulační kružnice elipsy s poloosami $a;b$ v jejím ve vedlejším vrcholu:

$$R = \frac{a^2}{b}$$

$$\left.\begin{aligned} a^2 &= r^2 + v_0^2 \\ b &= r \end{aligned}\right\} \Rightarrow R = r$$

## Šroubovice v Mongeově promítání:



## Šroubovice v pravoúhlé axonometrii: