M2 2010/2011

1. Draw a domain of function $f(x,y)=\sqrt{\sin(\pi(x^2+y^2))}$.

2. Draw a domain of function $f(x,y,z)=\sqrt{1-\sqrt{x^2+y^2}-z}+\sqrt{1-\sqrt{x^2+y^2}+z}$

3. Draw a graph of function $f(x,y)=\sqrt{x^2+y^2}$

4. Write the first order partial derivatives of the following functions
   $f(x,y,z)=\mathrm{e}^{\frac{x}{y}}+x^y$,
   $f(x,y,z)=\sqrt{xy}(3x+2z)\sqrt{yz}$.

5. Write the second order partial derivatives of function f at the point A:

   $f(x,y)=\arctan\frac{x-y}{x+y},\quad A=[3,1]$,
   $f(x,y)=(1+\log_y x)^3,\quad A=[\mathrm{e},\mathrm{e}]$.

6. Given functions $z=\sqrt{x^2+y^2}$ and $z=x-3y+\sqrt{3xy}$ find the angle between their gradients at the point [3,4].

7. Find the direction derivative of function $f=x^3-2x^2y+xy^2+1$ at the point $M=[1,2]$ along the vector $\overrightarrow{MN}$, where $N=[4,6]$.

8. Find the direction derivative of function $f=\mathrm{e}^{x^2+y^2}$ at the point [1,1] in direction of a vector (2,1).

9. Determine the equation of the tangent plane to a function $z=2x^2+y^2$ at the point [1,1,?].

10. Determine the equation of the tangent plane and normal line to a function $z=\sqrt{x^2+y^2}-xy$ at the point [3,4,-7].

11. Find order 3 Taylor's polynomial of of a function $f(x,y)=\frac{x}{y}$ at the point [1,1].

12. Determine the order 3 Taylor's polynomial at point (0,0) of a function $f(x,y)=\sin x\cos y$.

13. Determine local extremals of the following functions:
    $f(x,y)=x^3+xy^2-2xy-5x$,
    $f(x,y,z)=x^3+y^2+z^2+12xy+2z$,
    $f(x,y,z)=2x^4+y^4-x^2-2y^2$.

14. Find the constrained extremals of functions:
    $f(x,y,z)=6-4x-3y,$ satisfying $x^2+y^2=1$,
    $f(x,y)=\ln(xy),$ satisfying $x^2+y^2=2$,
    $f(x,y)=6x+6y$ satisfying $x^3+y^3=16$.

15. Determine the global extremals of the following functions within the given set M:
$f(x,y) = x^2 - xy + y^2,\ M : |x| + |y| \leq 1,$
$f(x,y) = y^2 - 2y - \mathrm{e}^{-x^2},\ M$ is a square with vertices [-1,0],[1,0],[1,2],[-1,2].

16. Calculate the following double integrals:

a) $\iint\limits_{\Omega} \mathrm{e}^{\frac{x}{y}} \mathrm{d}x\mathrm{d}y$ where Ω is given by $x = 0,\ y = 1,\ y = 2,\ y^2 = x,$

b) $\iint\limits_{\Omega} \frac{x}{y^2} \mathrm{d}x\mathrm{d}y$ where Ω is given by $x = 0,\ y = 1,\ y = 2,\ y^2 = x,$

c) $\iint\limits_{\Omega} \frac{x^2}{y^2} \mathrm{d}x\mathrm{d}y$, where Ω is given by $x = 2,\ y = x,\ xy = 1,$

d) $\iint\limits_{\Omega} \frac{\ln(x^2+y^2)}{x^2+y^2} \mathrm{d}x\mathrm{d}y$, where Ω is given by $1 \leq x^2 + y^2 \leq \mathrm{e}$.

17. Calculate the following triple integrals:

a) $\iiint\limits_{\Omega} \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $z = 1,\ z = 1 - x^2 - y^2,$

b) $\iiint\limits_{\Omega} x^2 \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $z = 0,\ z = 2,\ x^2 + y^2 = 1,$

c) $\iiint\limits_{\Omega} z \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $y = 4,\ z = 0,\ z = 3,\ x^2 - y = 0,$

d) $\iiint\limits_{\Omega} z\sqrt{x^2 + y^2} \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $z = 0,\ z = 3,\ y \geq 0,$
$x^2 + y^2 - 2x = 0,$

e) $\iiint\limits_{\Omega} \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $x^2 + 4y^2 + z^2 \leq 4$.

18. By transformation into polar coordinates calculate the following double integrals:

a) $\iint\limits_{\Omega} x^2 + y^2 \mathrm{d}x\mathrm{d}y$ where Ω is given by $1 \leq x^2 + y^2 \leq 4,\ |x| \leq y,$

b) $\iint\limits_{\Omega} x^2 \mathrm{d}x\mathrm{d}y$ where Ω is given by $0 \leq 2y \leq x,\ x^2 + 4y^2 \leq 4,$

c) $\iint\limits_{\Omega} \mathrm{arctg}\frac{y}{x} \mathrm{d}x\mathrm{d}y$ where Ω is given by $x > 0,\ \frac{\sqrt{3}}{3}x \leq y \leq \sqrt{3}x,\ 1 \leq x^2 + y^2 \leq 9.$

19. By transformation into cylindric coordinates calculate the following triple integrals:

a) $\iiint\limits_{\Omega} z \mathrm{d}x\mathrm{d}y\mathrm{d}z$ where Ω is given by $0 \leq z \leq 4 - \sqrt{x^2 + y^2},$

b) $\iiint\limits_{\Omega} z\sqrt{x^2 + y^2} \mathrm{d}x\mathrm{d}y\mathrm{d}z$ where Ω is given by $z = 0,\ z = 3,\ y \geq 0,\ x^2 + y^2 - 2x = 0.$

20. By transformation into sphere coordinates calculate the following triple integrals:

a) $\iiint\limits_{\Omega} \frac{1}{(x^2+y^2+z^2)^3} \mathrm{d}x\mathrm{d}y\mathrm{d}z$ where Ω is given by $1 \leq x^2 + y^2 + z^2 \leq 4,$

b) $\iiint\limits_{\Omega} \mathrm{d}x\mathrm{d}y\mathrm{d}z$ where Ω is given by $x^2 + 4y^2 + z^2 \leq 4,$

c) $\iiint\limits_{\Omega} \sqrt{(x^2 + y^2)} \mathrm{d}x\mathrm{d}y\mathrm{d}z$ where Ω is given by $x^2 + y^2 + z^2 \leq z,$

21. Calculate $\iiint\limits_{\Omega} z \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $y = 4,\ z = 0,\ z = 3,\ x^2 - y = 0.$

22. Calculate $\iiint\limits_{\Omega} z\sqrt{x^2 + y^2}\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $z = 0,\ z = 3,\ y \geq 0,\ x^2 + y^2 - 2x = 0.$

23. Calculate $\iiint\limits_{\Omega} \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where Ω is given by $x^2 + 4y^2 + z^2 \leq 4.$

24. Determine the curve integral $\int\limits_{\Gamma} (2z - \sqrt{x^2 + y^2})\mathrm{d}s$ over a spatial curve Γ given by parametrization

$$\begin{aligned} x &= t\cos t, \\ y &= t\sin t, \\ z &= t, \quad t \in \langle 0, 2\pi \rangle. \end{aligned}$$

25. Determine the second type curve integral $\int\limits_{\Gamma} y\mathrm{d}x + z\mathrm{d}y + x\mathrm{d}z$ over a spatial curve Γ given by

$$\Gamma = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 = 1,\ x + z = 1\}.$$

The orientation of Γ is clockwise. Draw the curve and derive its following parametrization

$$\begin{aligned} x &= \cos t, \\ y &= \sin t, \\ z &= 1 - \cos t, \quad t \in \langle 0, 2\pi \rangle. \end{aligned}$$

26. Determine the surface integral $\iint\limits_{S} (2z - \sqrt{x^2 + y^2})\mathrm{d}s$ over a surface S given by

$$S = \{[x, y, z] \in \mathbb{R}^3 : z = 2x^2 + 2y^2,\ z \leq 4,\ y \geq 0\}.$$

Use the following parametrization

$$\begin{aligned} x &= t, \\ y &= s, \\ z &= 2t^2 + 2s^2 \end{aligned}$$

and find the range of t and s.