

```

using System.IO;
using rat = MMP.RationalNumbers.Rational;
using rpoly = MMP.Tools.Rational.Polynomial;
using static MMP.Tools.Rational.Polynomial;
using static MMP.Tools.General;
using static System.Console;
using static System.Math;
using vec = MathNet.Numerics.LinearAlgebra.Vector<double>;
using mat = MathNet.Numerics.LinearAlgebra.Matrix<double>;
using MathNet.Numerics.LinearAlgebra.Double;

```

```

namespace MMP

```

```

{

```

```

    public static class ODE

```

```

    {

```

```

        public static void CodeGeneration()

```

```

        {

```

```

            int kmax = 12;

```

```

            var AB = new rat[kmax - 1][];

```

```

            var AM = new rat[kmax - 1][];

```

```

            for (int k = 1; k < kmax; k++)

```

```

            {

```

```

                AB[k - 1] = new rat[k];

```

```

                for (int i = 0; i < k; i++)

```

```

                {

```

```

                    rpoly la = Monom(0);

```

```

                    for (int j = 0; j < k; j++)

```

```

                        if (i != j)

```

```

                            la *= LinearFactor(-j);

```

```

                    la = la / la.Eval(-i);

```

```

        AB[k - 1][i] = Integral(la).Eval(1);
    }
    AM[k - 1] = new rat[k];
    for (int i = 0; i < k; i++)
    {
        rpoly la = Monom(0);
        for (int j = 0; j < k; j++)
            if (i != j)
                la *= LinearFactor(1 - j);
        la = la / la.Eval(1 - i);
        AM[k - 1][i] = Integral(la).Eval(1);
    }
}

using (StreamWriter outputFile = new StreamWriter("Adams.cs"))
{
    outputFile.WriteLine("using System;");
    outputFile.WriteLine();
    outputFile.WriteLine("public class Adams");
    outputFile.WriteLine("{");
    outputFile.WriteLine(ToCode<rat, double>("Bashforth", AB));
    outputFile.WriteLine(ToCode<rat, double>("Moulton", AM));
    outputFile.WriteLine("}");
}

}

public static vec f(double t, vec y)
{
    return new DenseVector(new double[] { y[1], -y[0] });
}

```

```

public static void sys()
{
    vec y0 = new DenseVector(2);
    y0[0] = 1;
    Adams am = new Adams(f, y0, 0, PI / 50, 11);
    (vec t, mat Y) = am.Solution(PI / 2);
    WriteLine(t);
    WriteLine(Y.Transpose().ToString("G"));
    WriteLine(t.Map(x => Cos(x)).ToString("G"));
}

public static void single()
{
    vec y0 = new DenseVector(1);
    y0[0] = 1;
    Adams am = new Adams((x, y) => y, y0, 0, 1.0 / 2, 11);
    (vec t, mat Y) = am.Solution(1.01);
    WriteLine(t);
    WriteLine(Y.Transpose().ToString("G"));
    WriteLine(t.Map(x => Exp(x)).ToString("G"));
}

static void Main()
{
    sys();
}

```

```

}

```

```

using System;
using System.Text;

namespace MMP.Tools
{
    public static class General
    {
        public static void Swap<T>(ref T a, ref T b)
        {
            T tmp;
            tmp = a;
            a = b;
            b = tmp;
        }

        public static string Print<T>(T a, int p, int n) where T : IComparable<T>
        {
            string s = a.ToString().Replace("-", "");

            if (p > 0 && s == "1") s = "";
            if (p <= n)
                if (a.CompareTo(default(T)) < 0)
                    s = "-" + s;
                else if (p != n)
                    s = "+" + s;
            if (p > 1)
                return s + string.Format($"x^{p}");
            else if (p == 1)
                return s + "x";
            else
                return s;
        }
    }
}

```

```
}
```

```
public static string Tab(int count = 1, string tab = "  ")
```

```
{
```

```
    string s = "";
```

```
    for (int i = 0; i < count; i++)
```

```
    {
```

```
        s += tab;
```

```
    }
```

```
    return s;
```

```
}
```

```
public static string ToCode<T, Tn>(string name, T[][] array)
```

```
{
```

```
    StringBuilder sb = new StringBuilder();
```

```
    sb.AppendFormat($"{Tab(1)}public static {typeof(Tn).Name}[][] {name} = ");
```

```
    sb.AppendLine();
```

```
    sb.AppendLine($"{Tab(1)}{{");
```

```
    for (int i = 0; i < array.Length; i++)
```

```
    {
```

```
        sb.Append($"{Tab(2)}new {typeof(Tn).Name}[] {{ ");
```

```
        for (int j = 0; j < array[i].Length - 1; j++)
```

```
        {
```

```
            sb.Append($"{array[i][j]}, ");
```

```
        }
```

```
        sb.Append($"{array[i][array[i].Length - 1]} ");
```

```
        if (i < array.Length - 1)
```

```
            sb.AppendLine("},");
```

```
        else
```

```
            sb.AppendLine("}");
```

```
    }
```

```

        sb.AppendLine($"{Tab(1)}}};");
        return sb.ToString().Replace("/", ".0/");
    }
}

```

```

using System;
using System.Collections.Generic;
using static System.Math;
using MathNet.Numerics.LinearAlgebra.Double;
using vec = MathNet.Numerics.LinearAlgebra.Vector<double>;
using mat = MathNet.Numerics.LinearAlgebra.Matrix<double>;

public class Adams
{
    public static Double[][] Bashforth =
    {
        new Double[] { 1 },
        new Double[] { 3.0/2, -1.0/2 },
        new Double[] { 23.0/12, -4.0/3, 5.0/12 },
        new Double[] { 55.0/24, -59.0/24, 37.0/24, -3.0/8 },
        new Double[] { 1901.0/720, -1387.0/360, 109.0/30, -637.0/360, 251.0/720 },
        new Double[] { 4277.0/1440, -2641.0/480, 4991.0/720, -3649.0/720, 959.0/480, -95.0/288 },
        new Double[] { 198721.0/60480, -18637.0/2520, 235183.0/20160, -10754.0/945, 135713.0/20160, -
5603.0/2520, 19087.0/60480 },
        new Double[] { 16083.0/4480, -1152169.0/120960, 242653.0/13440, -296053.0/13440,
2102243.0/120960, -115747.0/13440, 32863.0/13440, -5257.0/17280 },
    }
}

```

```

        new Double[] { 14097247.0/3628800, -21562603.0/1814400, 47738393.0/1814400, -
69927631.0/1814400, 862303.0/22680, -45586321.0/1814400, 19416743.0/1814400, -4832053.0/1814400,
1070017.0/3628800 },
        new Double[] { 4325321.0/1036800, -104995189.0/7257600, 6648317.0/181440, -28416361.0/453600,
269181919.0/3628800, -222386081.0/3628800, 15788639.0/453600, -2357683.0/181440, 20884811.0/7257600,
-25713.0/89600 },
        new Double[] { 2132509567.0/479001600, -2067948781.0/119750400, 1572737587.0/31933440, -
1921376209.0/19958400, 3539798831.0/26611200, -82260679.0/623700, 2492064913.0/26611200, -
186080291.0/3991680, 2472634817.0/159667200, -52841941.0/17107200, 26842253.0/95800320 }
    };

    public static Double[][] Moulton =
    {
        new Double[] { 1 },
        new Double[] { 1.0/2, 1.0/2 },
        new Double[] { 5.0/12, 2.0/3, -1.0/12 },
        new Double[] { 3.0/8, 19.0/24, -5.0/24, 1.0/24 },
        new Double[] { 251.0/720, 323.0/360, -11.0/30, 53.0/360, -19.0/720 },
        new Double[] { 95.0/288, 1427.0/1440, -133.0/240, 241.0/720, -173.0/1440, 3.0/160 },
        new Double[] { 19087.0/60480, 2713.0/2520, -15487.0/20160, 586.0/945, -6737.0/20160,
263.0/2520, -863.0/60480 },
        new Double[] { 5257.0/17280, 139849.0/120960, -4511.0/4480, 123133.0/120960, -88547.0/120960,
1537.0/4480, -11351.0/120960, 275.0/24192 },
        new Double[] { 1070017.0/3628800, 2233547.0/1814400, -2302297.0/1814400, 2797679.0/1814400, -
31457.0/22680, 1573169.0/1814400, -645607.0/1814400, 156437.0/1814400, -33953.0/3628800 },
        new Double[] { 25713.0/89600, 9449717.0/7257600, -1408913.0/907200, 200029.0/90720, -
8641823.0/3628800, 6755041.0/3628800, -462127.0/453600, 335983.0/907200, -116687.0/1451520,
8183.0/1036800 },
        new Double[] { 26842253.0/95800320, 164046413.0/119750400, -296725183.0/159667200,
12051709.0/3991680, -33765029.0/8870400, 2227571.0/623700, -21677723.0/8870400, 23643791.0/19958400,
-12318413.0/31933440, 9071219.0/119750400, -3250433.0/479001600 }
    }

```

```
};
```

```
Func<double, vec, vec> f;  
public List<vec> Y = new List<vec>();  
public List<double> t = new List<double>();  
int order;  
double tau;
```

```
public Adams(Func<double, vec, vec> f, vec y, double t0, double tau, int ord = 6)  
{  
    this.f = f;  
    Y.Add(y); t.Add(t0);  
    if (ord > Bashforth.Length + 1)  
        throw new IndexOutOfRangeException();  
    double dt = Pow(tau, ord);  
    int it = (int)Ceiling(1 / tau);  
    vec fn;  
    for (int k = 1; k < ord; k++)  
    {  
        int n = it * k - Y.Count + 1;  
        for (int j = 0; j < n; j++)  
        {  
            fn = new DenseVector(y.Count);  
            int end = Y.Count - 1;  
            for (int i = 0; i < k; i++)  
                fn += f(t[end - i], Y[end - i]) * Bashforth[k - 1][i];  
            Y.Add(Y[Y.Count - 1] + dt * fn);  
            t.Add(t[t.Count - 1] + dt);  
        }  
        for (int j = 0; j < k; j++)  
        {
```



```

        Y.RemoveRange(j + 1, it - 1);
        t.RemoveRange(j + 1, it - 1);
    }
    dt *= it;
}
order = ord;
this.tau = tau;
}

public void Step()
{
    vec fn = new DenseVector(Y[0].Count);
    int end = Y.Count - 1;
    for (int i = 0; i < order; i++)
        fn += f(t[end - i], Y[end - i]) * Bashforth[order - 1][i];
    Y.Add(Y[Y.Count - 1] + tau * fn);
    t.Add(t[t.Count - 1] + tau);
}

public (vec, mat) Solution(double T)
{
    while (t[t.Count - 1] + tau <= T)
    {
        Step();
    }
    vec times = DenseVector.OfEnumerable(t);
    mat values = DenseMatrix.OfColumns(Y);
    return (times, values);
}
}

```