**<u>Colouring a graph containing no odd circles</u>**

Let $G = (N, E)$ be such a graph.

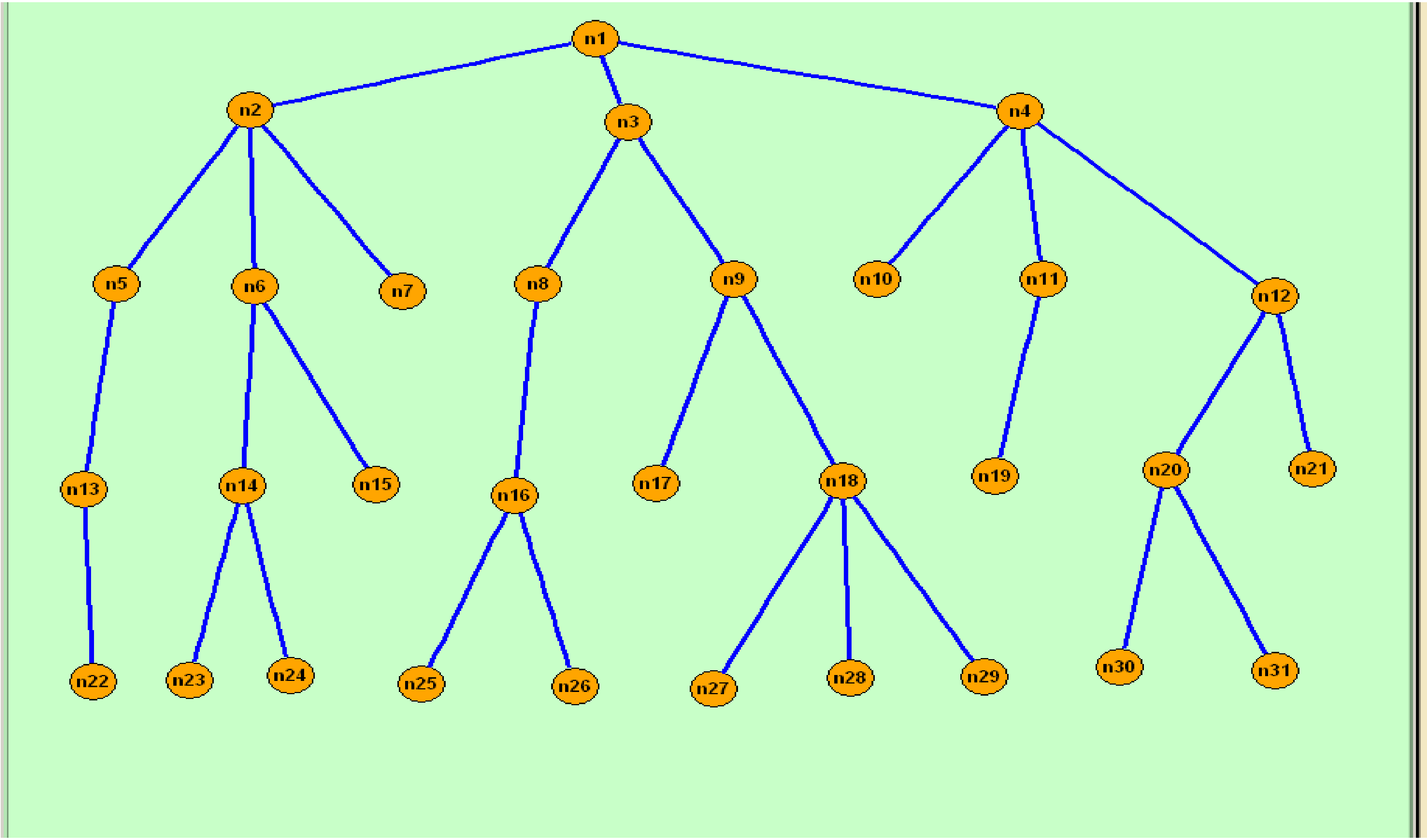
Start from any node $v_1$. Assign $v_1$ level 0.

Assign to all nodes adjacent to $v_1$ level 1.

Partition the set of nodes of $G$ into sets of nodes at different levels.

Let $v_{i_1}, v_{i_2}, \ldots, v_{i_r},$ be the nodes at level $i$. Consider all nodes adjacent to $v_{i_1}$ that are not assigned levels 0, 1, 2, ..., $i$. Assign these level $i + 1$.

Continue this process until all nodes are assigned a level. Assign now one colour to nodes at odd levels and another colour to all those at an even level.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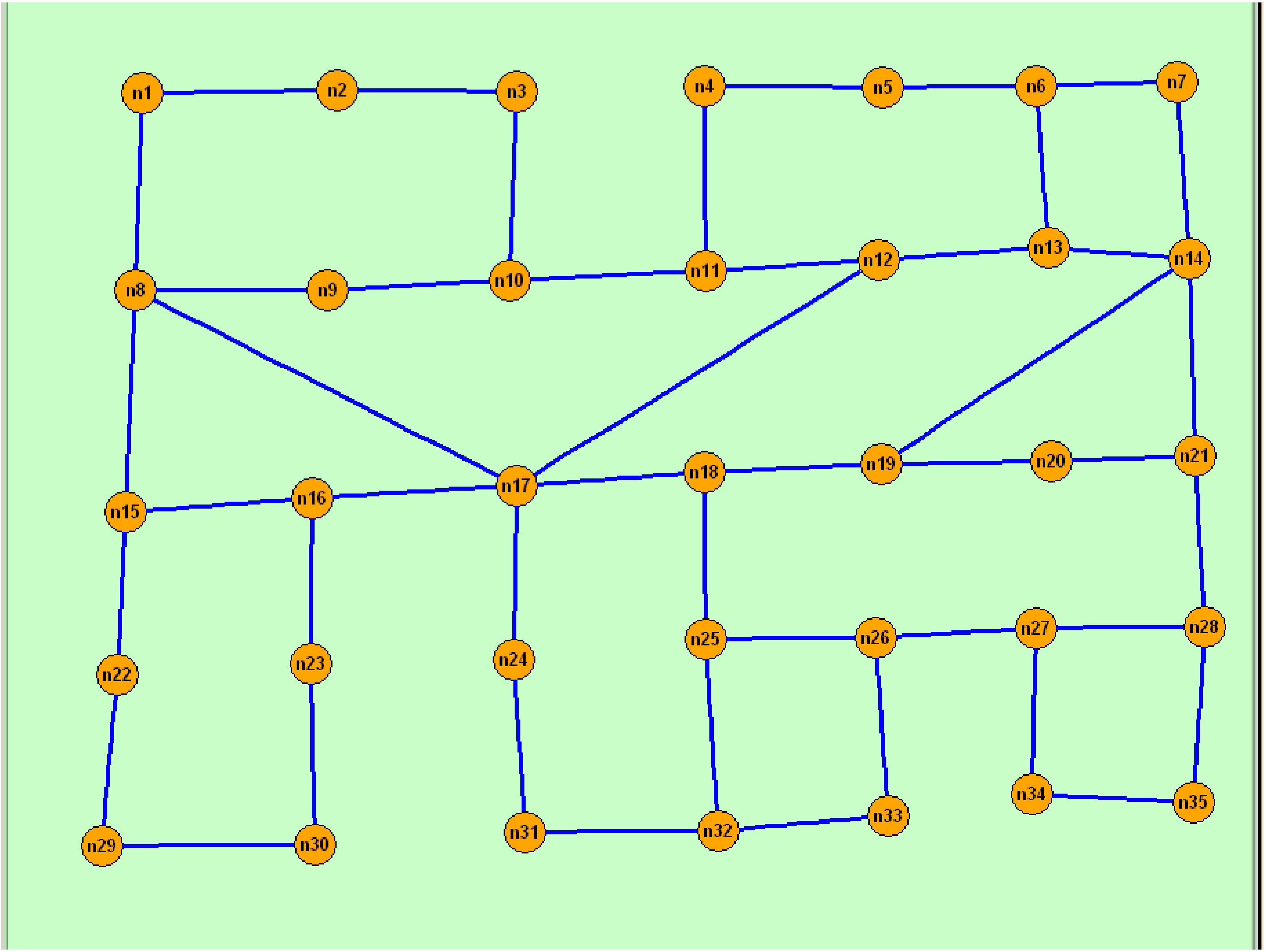
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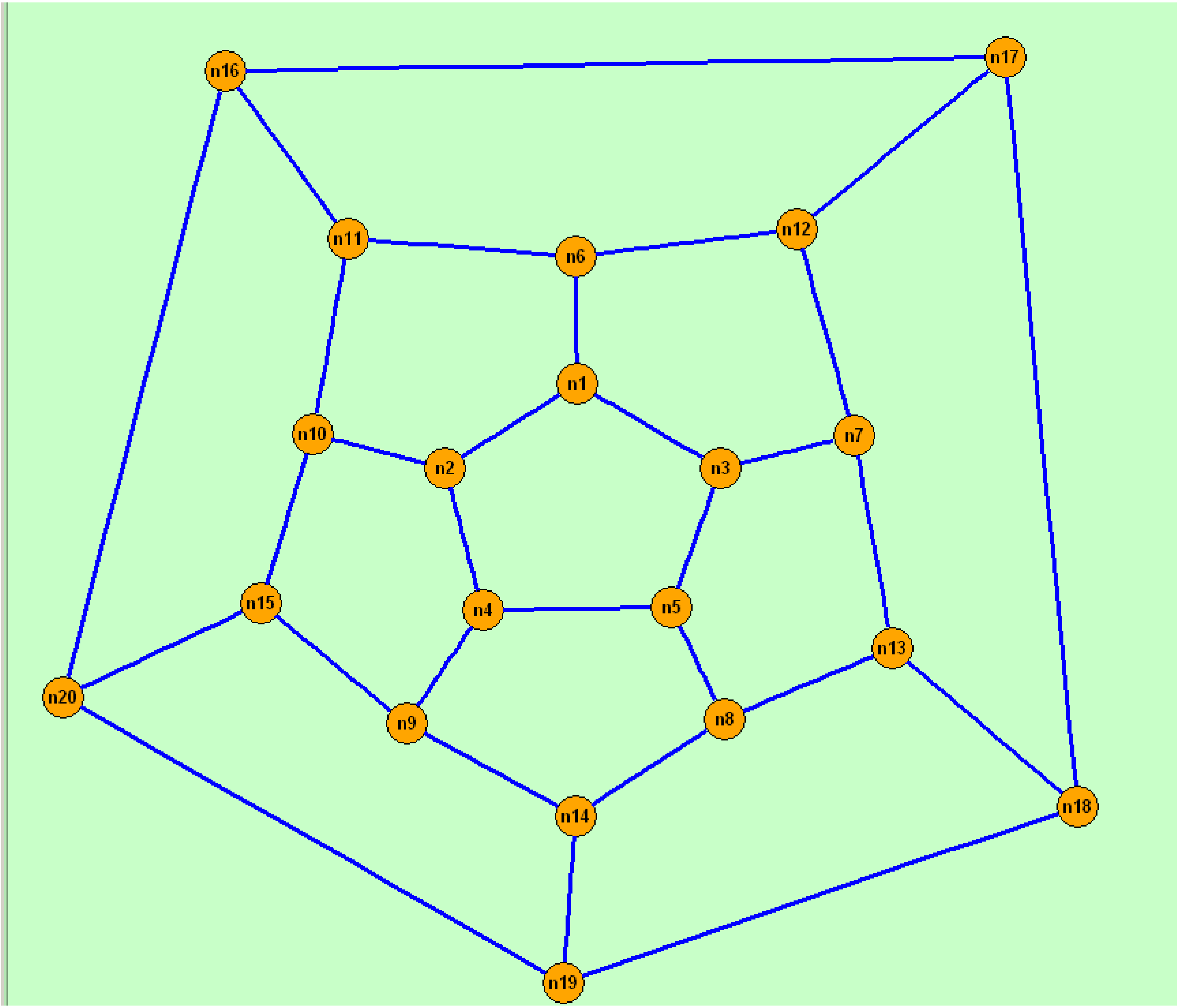
## Approximation Algorithm

No simple characterization is known for graphs that are *k*-colourable if $k > 2$. In general, it is a hard problem to determine the chromatic number of an arbitrary graph. No algorithm is known that finds a colouring pattern using the fewest possible colours for every input graph. However, there are colouring algorithms that can be used to approximate the best colouring in the sense that they may sometimes use more colours than necessary. The following is an algorithm known as the ***largest-first algorithm:***

1. order the nodes by their degrees creating a sequence: $\boldsymbol{v} = v_{1,} v_{2,} \ldots$ and consider a sequence $\boldsymbol{c} = c_{1,} c_{2,} \ldots$ of colours
2. assign $c_1$ to $v_1$ and keep assigning $c_1$ to nodes in $\boldsymbol{v}$ non adjacent to $v_1$ and previously coloured nodes until you can
3. assign $c_2$ to the node $v_i$ with the greatest degree in $\boldsymbol{v}$ that has not been assigned a colour and then continue assigning $c_2$ to others nodes in $\boldsymbol{v}$ non-adjacent to $v_i$ until you can
4. continue this process until all the nodes have been assigned colours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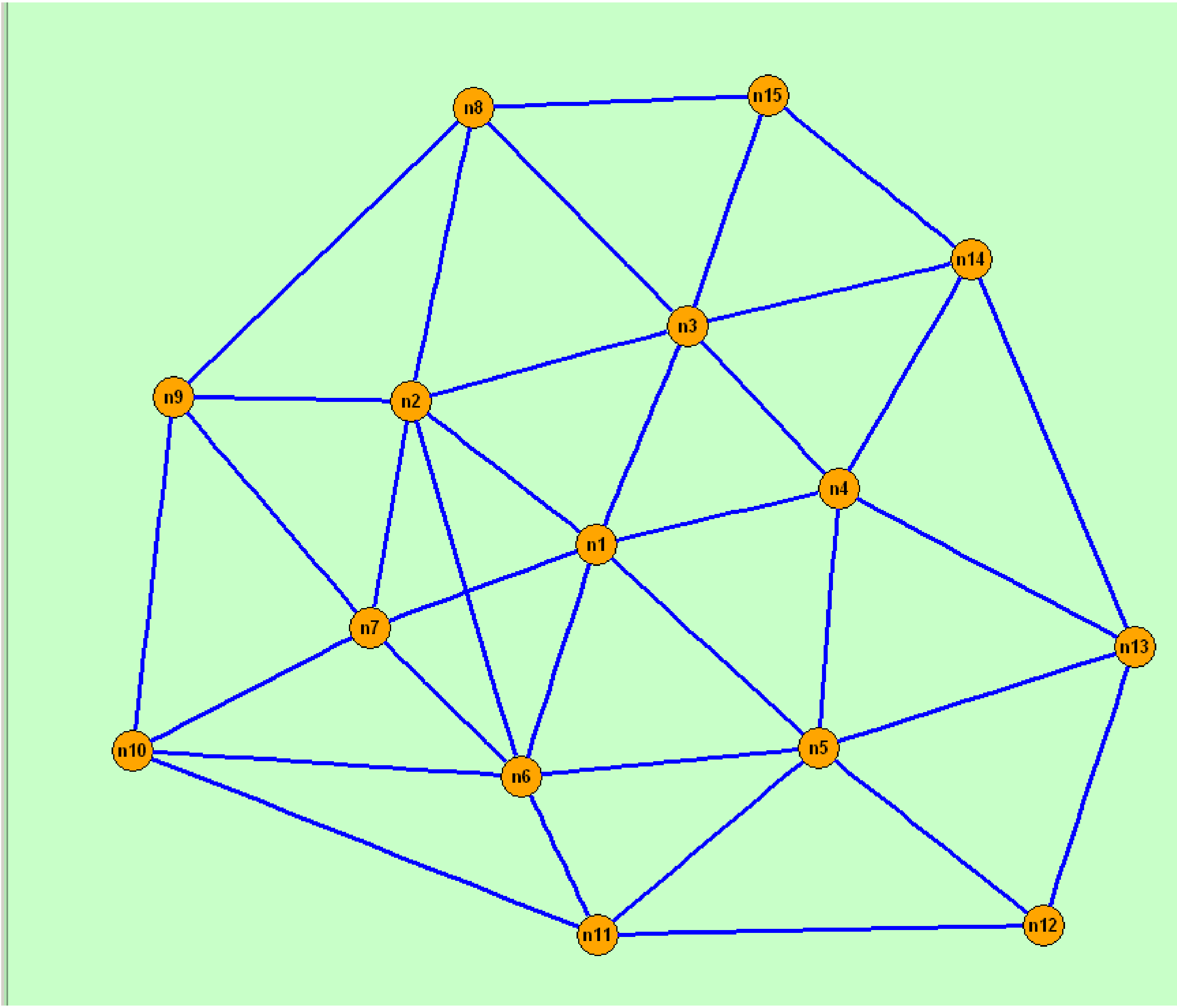

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