M1 EXAMPLES

Determine the primitive function (integrate):

1. Draw a graph, determine the domain and mark the intersections with the axes:
   a) $f(x) = |1 - \mathrm{e}^{(1-x)}|$
   b) $f(x) = 1 + 2\mathrm{arctg}(x - 2)$
   c) $f(x) = 2\sin(2x - \frac{\pi}{4})$

2. Determine the scalar (dot) product and vector (cross) product of the following vectors: $\vec{u} = (1, -1, 2)$, $\vec{v} = (3, 1, 1)$. Determine also a norm of both vectors and the angle between them.

3. Using the adjoint matrix find an inverse and **check the result**:

$$\begin{pmatrix} 2 & 1 & -3 \\ 4 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

4. Multiply the following matrices:

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 1 \\ 1 & 5 & 3 \end{pmatrix}, \; B = \begin{pmatrix} 3 & 7 \\ 2 & -6 \\ 1 & 1 \end{pmatrix}.$$

   Check that $A \cdot B \neq B \cdot A$, especially check whether both sides of this inequality exist.

5. Count the determinant of a matrix:

$$\begin{pmatrix} 1 & 2 & -3 \\ 4 & -0 & 1 \\ 1 & 5 & 3 \end{pmatrix}$$

6. Count the determinant of a matrix:

$$\begin{pmatrix} 1 & 2 & -1 & 0 \\ 4 & 0 & 1 & 2 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 1 & 0 \end{pmatrix}$$

7. Determine a rank of system matrix and solve the system

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 3 & -2 & 1 & 0 \\ -1 & 2 & 4 & -2 \\ 2 & -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

8. Write the set of solutions (if there are any) of the system of linear equations. Discuss the number of solutions. The system is already after the Gauss elimination.

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & -2 & 1 & 2 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

9. Given a plane

$$\begin{aligned} x &= 1 - 2t + 4s \\ y &= t + s \\ z &= 5 - 3t + s \end{aligned}$$

write the coordinates of one of its points and determine two direction vectors. Find the normal vector of this plane and its general equation.

10. Determine the angle of two planes $\delta:\ x-y+\sqrt{2}z+2=0$ and $\sigma:\ x+y+\sqrt{2}z-3=0$. Note: angle of two planes = angle of their normal vectors.

11. Draw graphs of the following functions (just shift and rollover the basic graph shape):
    a) $f(x) = 2 - 3^{-(x-1)}$,
    b) $f(x) = |(\frac{1}{3})^{2-x} - 1|$,
    c) $f(x) = \log_{\frac{1}{3}} |x+2| - 1$,
    d) $f(x) = |\log_{\pi}(3-x) - 5|$,
    e) $f(x) = \sqrt[3]{(x-1)^2} + 5$,
    f) $f(x) = \frac{2x+3}{x+1}$,
    g) $f(x) = -1 - \frac{1}{(x+2)^2}$
    h) $f(x) = 1 + 2\sin(x - \frac{\pi}{2})$,
    i) $f(x) = -\text{cotg}(\frac{\pi}{4} - x) + 2$,
    j) $f(x) = |\frac{\pi}{2} - \arcsin(x-3)|$,
    k) $f(x) = \text{arctg}(x-2) - 1$.

12. Determine the domain of the following functions:
    a) $f(x) = \sqrt{3x - x^3}$,
    b) $f(x) = \ln(1 - e^x)$,
    c) $f(x) = \arccos(2x - 1)$,
    d) $f(x) = \sqrt{\frac{1-x}{1+x}}$.

13. Write a derivative of following functions:
    a) $f(x) = \cos x \cdot \ln x$,
    b) $f(x) = \sin^2 x \cdot \sqrt{x}$,
    c) $f(x) = \frac{x^3+2x+1}{\text{tg} x}$,

d) $f(x) = \frac{\mathrm{arctg} x}{\mathrm{e}^{x^2}}$,

e) $f(x) = 2^{x \cdot \ln x}$,

f) $f(x) = \ln^3 \sin^2 x^2$,

g) $f(x) = \frac{x \cdot \mathrm{e}^x}{x^2 - \cos x}$,

h) $f(x) = \ln \frac{x+1}{x-1}$

i) $f(x) = \mathrm{arctg}(x^3) \cdot \sqrt[3]{\sin x^2}$

j) $f(x) = \cos(x^2 + 3x - 5)$,

k) $f(x) = \frac{\mathrm{arctg}(\ln x)}{x^2+1}$.

14. Determine the following limits:

a) $\lim_{x \to \frac{\pi}{4}} \frac{\mathrm{tg} x - 1}{\sin 4x}$,

b) $\lim_{x \to 0} \frac{\mathrm{e}^x - \mathrm{e}^{-x} - 2x}{x^3}$,

c) $\lim_{x \to 0+} x^{\frac{3}{4 + \ln x}}$,

d) $\lim_{x \to 0} \frac{\mathrm{arctg} 2x}{\sin 3x}$,

e) $\lim_{x \to 1} (1 - x) \mathrm{tg} \frac{\pi x}{2}$,

f) $\lim_{x \to \frac{\pi}{2}} \left(\frac{x}{\mathrm{cotg} x} - \frac{\pi}{2 \cos x} \right)$

15. Using derivatives sketch graphs of the following functions (determine the domain, intervals of monotony, shape, asymptotes)

a) $f(x) = \frac{2x}{x^2+1}$

b) $f(x) = x \cdot \mathrm{e}^{\frac{1}{x}}$

c) $f(x) = \frac{\ln(x-1)}{x-1}$

d) $f(x) = x + 2\mathrm{arccotg} x$

e) $f(x) = \frac{1-x^3}{x^2}$

f) $f(x) = x \cdot \mathrm{e}^{-\frac{x^2}{2}}$

g) $f(x) = \sqrt[3]{1 - x^3}$

16. Using derivatives (L'Hospital's rule) determine the following limits:

a) $\lim_{x \to 0} \frac{\mathrm{e}^x - \mathrm{e}^{-x} - 2x}{x - \sin x}$

b) $\lim_{x \to \infty} (\pi - 2\mathrm{arctg} x) \ln x$

c) $\lim_{x \to 0+} \left(\frac{1}{x} \right)^{\mathrm{tg} x}$

d) $\lim_{x \to 0+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

e) $\lim_{x \to 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3}$

f) $\lim_{x \to 0} (\cos 2x)^{\frac{3}{x^2}}$

g) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\ln(x+1)} \right)$

17. Determine degree n Taylor's polynomial of function $f(x)$ at the point x_0:

a) $f(x) = \ln x,\ n = 4,\ x_0 = 4$

b) $f(x) = x\mathrm{e}^{-x},\ n = 4,\ x_0 = 0$

18. Integrate

(a) By parts (per partes)

a) $\int (x^2 + 2x + 17)\mathrm{e}^x \mathrm{d}x$

b) $\int x \ln x \mathrm{d}x$

c) $\int x^2 \mathrm{arctg} \mathrm{d}x$

d) $\int x\mathrm{e}^{-x} \mathrm{d}x$

e) $\int \mathrm{e}^x \cos x \mathrm{d}x$

f) $\int \ln x \mathrm{d}x$

(b) By substitution

a) $\int \frac{\sin^2 x}{\cos^4 x} \mathrm{d}x$

b) $\int \frac{\sin x}{\sqrt[3]{1+2\cos x}} \mathrm{d}x$

c) $\int \frac{1}{\sqrt{4x+9}} \mathrm{d}x$

d) $\int \mathrm{e}^x \cos(\mathrm{e}^x) \mathrm{d}x$

e) $\int \sin^7 x \cos x \mathrm{d}x$

(c) By decomposition into partial fractions

a) $\int \frac{x^4+6x^2+x-2}{x^4-2x} \mathrm{d}x$

b) $\int \frac{x}{(x-1)(x+1)^2} \mathrm{d}x$

c) $\int \frac{8x-31}{x^2-9x+14} \mathrm{d}x$

(d) Choose the proper method of integration and integrate

a) $\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}} \mathrm{d}x$

b) $\int \mathrm{arctg} x \mathrm{d}x$

c) $\int \frac{6x+6}{2x^2+3x} \mathrm{d}x$

d) $\int \sin^5 x \mathrm{d}x$