

PŘÍKLADY NA DERIVACI FUNKCE JEDNÉ PROMĚNNÉ 1

Derivujte a neupravujte:

ZADÁNÍ

a) $f(x) = \ln(x^2)$

b) $f(x) = \sin(e^x)$

c) $f(x) = \left(\frac{2}{\arctg(e^x)} - 4\right) \cdot \ln(x^4)$

d) $f(x) = \sin(\cos x)$

e) $f(x) = \frac{\arctg(x^2)}{8 \cdot (x-6) + \cos(x^2)}$

f) $f(x) = 3 \cdot \left(\sqrt{\arctg x} + \frac{9}{x} - 4 + e^{x^3}\right)$

g) $f(x) = e^{\arctg x} \cdot e^{\sqrt{x}}$

h) $f(x) = \sin\left(\frac{1}{x^2}\right) \cdot \left(x - \frac{9}{\sin(x^2)}\right)$ $f'(x) = \cos\left(\frac{1}{x^2}\right) \cdot \frac{-2}{x^3} \cdot \left(x - \frac{9}{\sin(x^2)}\right) + \sin\left(\frac{1}{x^2}\right) \cdot \left(1 - \frac{-9 \cdot \cos(x^2) \cdot 2x}{(\sin(x^2))^2}\right)$

i) $f(x) = \cos\left(\frac{1}{x^4}\right) \cdot (\sin(e^x) + 6)$ $f'(x) = \left(-\sin\left(\frac{1}{x^4}\right)\right) \cdot \frac{-4}{x^5} \cdot (\sin(e^x) + 6) + \cos\left(\frac{1}{x^4}\right) \cdot (\cos(e^x) \cdot e^x)$

j) $f(x) = \ln(\cos x)$

k) $f(x) = (\sin x)^4$

l) $f(x) = \frac{\sin(\arctg x)}{\ln x}$

m) $f(x) = \frac{\arctg\left(\frac{1}{x^5}\right)}{5}$

n) $f(x) = \frac{1}{(\sin x)^2}$

o) $f(x) = e^{\cos x}$

p) $f(x) = \arctg(e^x)$

q) $f(x) = \frac{1}{(\ln x)^5}$

r) $f(x) = \sin(e^x)$

s) $f(x) = \sin(x^4)$

t) $f(x) = \ln(x^6)$

u) $f(x) = \frac{\arctg(\ln x)}{\sqrt{\ln x}}$

v) $f(x) = \frac{1}{(\ln x)^5}$

w) $f(x) = \ln(\sqrt{x})$

x) $f(x) = \arctg(\sin x)$

y) $f(x) = (\cos(\cos x) - 4) \cdot \sin(\sqrt{x})$

z) $f(x) = \frac{1}{(\arctg x)^4}$

ž) $f(x) = (e^{\sqrt{x}} - \sin(\sqrt{x})) \cdot 5$

ž) $f(x) = \sin(1 + \arctg(e^x))$

ŘEŠENÍ

$$f'(x) = \frac{1}{x^2} \cdot 2x$$

$$f'(x) = \cos(e^x) \cdot e^x$$

$$f'(x) = \left(\frac{-2 \cdot \frac{1}{1+(e^x)^2} \cdot e^x}{(\arctg(e^x))^2}\right) \cdot \ln(x^4) + \left(\frac{2}{\arctg(e^x)} - 4\right) \cdot \frac{1}{x^4} \cdot 4x^3$$

$$f'(x) = \cos(\cos x) \cdot (-\sin x)$$

$$f'(x) = \frac{\frac{1}{1+(x^2)^2} \cdot 2x \cdot (8 \cdot (x-6) + \cos(x^2)) - \arctg(x^2) \cdot (8 \cdot (1) + (-\sin(x^2)) \cdot 2x)}{(8 \cdot (x-6) + \cos(x^2))^2}$$

$$f'(x) = 3 \cdot \left(\frac{1}{2 \cdot \sqrt{\arctg x}} \cdot \frac{1}{1+x^2} + \frac{-9}{x^2} + e^{x^3} \cdot 3x^2\right)$$

$$f'(x) = e^{\arctg x} \cdot \frac{1}{1+x^2} \cdot e^{\sqrt{x}} + e^{\arctg x} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{\cos(\arctg x) \cdot \frac{1}{1+x^2} \cdot \ln x - \sin(\arctg x) \cdot \frac{1}{x}}{(\ln x)^2}$$

$$f'(x) = \frac{\frac{1}{1+\left(\frac{1}{x^5}\right)^2} \cdot \frac{-5}{x^6}}{5}$$

$$f'(x) = \frac{-2}{(\sin x)^3} \cdot \cos x$$

$$f'(x) = e^{\cos x} \cdot (-\sin x)$$

$$f'(x) = \frac{1}{1+(e^x)^2} \cdot e^x$$

$$f'(x) = \frac{-5}{(\ln x)^6} \cdot \frac{1}{x}$$

$$f'(x) = \cos(e^x) \cdot e^x$$

$$f'(x) = \cos(x^4) \cdot 4x^3$$

$$f'(x) = \frac{1}{x^6} \cdot 6x^5$$

$$f'(x) = \frac{\frac{1}{1+(\ln x)^2} \cdot \frac{1}{x} \cdot \sqrt{\ln x} - \arctg(\ln x) \cdot \frac{1}{2 \cdot \sqrt{\ln x}} \cdot \frac{1}{x}}{(\sqrt{\ln x})^2}$$

$$f'(x) = \frac{-5}{(\ln x)^6} \cdot \frac{1}{x}$$

$$f'(x) = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{1+(\sin x)^2} \cdot \cos x$$

$$f'(x) = \sin(\cos x) \cdot \sin x \cdot \sin(\sqrt{x}) + (\cos(\cos x) - 4) \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{-4}{(\arctg x)^5} \cdot \frac{1}{1+x^2}$$

$$f'(x) = \left(e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}\right) \cdot 5$$

$$f'(x) = \cos(1 + \arctg(e^x)) \cdot \left(\frac{1}{1+(e^x)^2} \cdot e^x\right)$$

PŘÍKLADY NA DERIVACI FUNKCE JEDNÉ PROMĚNNÉ 2

Derivujte a neupravujte:

ZADÁNÍ

a) $f(x) = \frac{1}{\arctg(\cos(x))} - 5$

b) $f(x) = (\sin(\ln(x)))^3 + 3$

c) $f(x) = e^{\cos(\sqrt{x})} - \sqrt{x}$

d) $f(x) = \frac{1}{\arctg(e^x)} + 3$

e) $f(x) = \left(\cos\left(\frac{1}{x^4}\right)\right)^4$

f) $f(x) = \sin(\arctg(x))$

g) $f(x) = e^{e^{\cos(x)}}$

h) $f(x) = \sin(\ln(\sin(x)))$

i) $f(x) = \ln\left(\cos\left(\frac{1}{x^4}\right)\right)$

j) $f(x) = \sqrt{\sin(e^x)}$

k) $f(x) = e^{\cos(\arctg(x))}$

l) $f(x) = \frac{1}{(\sin(\sin(x)))^5}$

m) $f(x) = \ln\left(\frac{1}{x^2}\right)$

n) $f(x) = \ln\left(\frac{1}{x^3}\right) + \cos(\sin(\sqrt{x}))$

o) $f(x) = \cos\left(\frac{1}{x^4}\right) \cdot (8x + 6)$

p) $f(x) = 6 \cdot \left(\frac{9}{8 - \cos(e^x)} + 6\right)$

q) $f(x) = \frac{\ln(\sin(x))}{\arctg(x)}$

r) $f(x) = \frac{\sin(\arctg(x))}{\ln x}$

s) $f(x) = \frac{1}{(\sin x)^2}$

t) $f(x) = \arctg\left(\frac{1}{x^2}\right)$

u) $f(x) = e^{\frac{1}{x^2}}$

v) $f(x) = 2 \cdot \ln(x^2)$

w) $f(x) = \arctg\left(\frac{1}{x^4}\right)$

x) $f(x) = \frac{1}{(\ln x)^5}$

y) $f(x) = \frac{1}{\ln x}$

z) $f(x) = \frac{x}{e^{e^{\sqrt{x}}} + \ln((\arctg(x))^2) + 3} \quad f'(x) = \frac{\left(e^{e^{\sqrt{x}}} + \ln((\arctg(x))^2) + 3\right) - x \cdot \left(e^{e^{\sqrt{x}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{(\arctg(x))^2} \cdot 2(\arctg(x)) \cdot \frac{1}{1+x^2}\right)}{\left(e^{e^{\sqrt{x}}} + \ln((\arctg(x))^2) + 3\right)^2}$

ž) $f(x) = \sin\left(\frac{1}{x^2}\right) \cdot \left(x - \frac{9}{\sin(x^2)}\right) \quad f'(x) = \cos\left(\frac{1}{x^2}\right) \cdot \frac{-2}{x^3} \cdot \left(x - \frac{9}{\sin(x^2)}\right) + \sin\left(\frac{1}{x^2}\right) \cdot \left(1 - \frac{-9 \cdot \cos(x^2) \cdot 2x}{(\sin(x^2))^2}\right)$

ž) $f(x) = (\cos(e^x))^{-4}$

ŘEŠENÍ

$$f'(x) = \frac{-1}{(\arctg(\cos(x)))^2} \cdot \frac{1}{1+(\cos(x))^2} \cdot (-\sin(x))$$

$$f'(x) = 3(\sin(\ln(x)))^2 \cdot \cos(\ln(x)) \cdot \frac{1}{x}$$

$$f'(x) = e^{\cos(\sqrt{x})} \cdot (-\sin(\sqrt{x})) \cdot \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{-1}{(\arctg(e^x))^2} \cdot \frac{1}{1+(e^x)^2} \cdot e^x$$

$$f'(x) = 4\left(\cos\left(\frac{1}{x^4}\right)\right)^3 \cdot \left(-\sin\left(\frac{1}{x^4}\right)\right) \cdot \frac{-4}{x^5}$$

$$f'(x) = \cos(\arctg(x)) \cdot \frac{1}{1+x^2}$$

$$f'(x) = e^{e^{\cos(x)}} \cdot e^{\cos(x)} \cdot (-\sin(x))$$

$$f'(x) = \cos(\ln(\sin(x))) \cdot \frac{1}{\sin(x)} \cdot \cos(x)$$

$$f'(x) = \frac{1}{\cos\left(\frac{1}{x^4}\right)} \cdot \left(-\sin\left(\frac{1}{x^4}\right)\right) \cdot \frac{-4}{x^5}$$

$$f'(x) = \frac{1}{2 \cdot \sqrt{\sin(e^x)}} \cdot \cos(e^x) \cdot e^x$$

$$f'(x) = e^{\cos(\arctg(x))} \cdot (-\sin(\arctg(x))) \cdot \frac{1}{1+x^2}$$

$$f'(x) = \frac{-5}{(\sin(\sin(x)))^6} \cdot \cos(\sin(x)) \cdot \cos(x)$$

$$f'(x) = \frac{1}{\frac{1}{x^2}} \cdot \frac{-2}{x^3}$$

$$f'(x) = \frac{1}{\frac{1}{x^3}} \cdot \frac{-3}{x^4} + (-\sin(\sin(\sqrt{x}))) \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$f'(x) = \left(-\sin\left(\frac{1}{x^4}\right)\right) \cdot \frac{-4}{x^5} \cdot (8x + 6) + \cos\left(\frac{1}{x^4}\right) \cdot 8$$

$$f'(x) = 6 \cdot \left(\frac{-9 \cdot (\sin(e^x)) \cdot e^x}{(8 - \cos(e^x))^2}\right)$$

$$f'(x) = \frac{\left(\frac{1}{\sin(x)} \cdot \cos(x)\right) \cdot \arctg(x) - \ln(\sin(x)) \cdot \frac{1}{1+x^2}}{(\arctg(x))^2}$$

$$f'(x) = \frac{\cos(\arctg(x)) \cdot \frac{1}{1+x^2} \cdot \ln x - \sin(\arctg(x)) \cdot \frac{1}{x}}{(\ln x)^2}$$

$$f'(x) = \frac{-2}{(\sin x)^3} \cdot \cos x$$

$$f'(x) = \frac{1}{1+\left(\frac{1}{x^2}\right)^2} \cdot \frac{-2}{x^3}$$

$$f'(x) = e^{\frac{1}{x^2}} \cdot \frac{-2}{x^3}$$

$$f'(x) = 2 \cdot \frac{1}{x^2} \cdot 2x$$

$$f'(x) = \frac{1}{1+\left(\frac{1}{x^4}\right)^2} \cdot \frac{-4}{x^5}$$

$$f'(x) = \frac{-5}{(\ln x)^6} \cdot \frac{1}{x}$$

$$f'(x) = \frac{-1}{(\ln x)^2} \cdot \frac{1}{x}$$

PŘÍKLADY NA DERIVACI FUNKCE JEDNÉ PROMĚNNÉ 3

Derivujte a neupravujte:

ZADÁNÍ

$$a) \quad f(x) = \frac{1}{\left(\frac{x^5+x}{\sin x + \arctg x}\right)^3}$$

$$b) \quad f(x) = \ln \left(\sqrt{\cos x \cdot \sqrt{x}} \right)$$

$$c) \quad f(x) = \sin \left(e^{e^{x^6}} \right)$$

$$d) \quad f(x) = \ln x + e^x - \frac{e^x}{x^3}$$

$$e) \quad f(x) = \cos \left((\sin x)^3 \right)$$

$$f) \quad f(x) = \frac{1}{\ln x} \cdot (\sin x - \cos x)$$

$$g) \quad f(x) = e^{x^{-2}} \cdot (\arctg x)^{-3} \quad f'(x) = \left(e^{x^{-2}} \cdot (-2x^{-3}) \cdot (\arctg x)^{-3} + e^{x^{-2}} \cdot (-3 (\arctg x)^{-4} \cdot \frac{1}{1+x^2}) \right)$$

$$h) \quad f(x) = \ln(x \cdot \arctg x)$$

$$i) \quad f(x) = \frac{3-x^{-3}}{\arctg x \cdot x^4}$$

$$j) \quad f(x) = \frac{e^x + \arctg x - \ln(\ln x)}{2 \cdot \sqrt{x}}$$

$$k) \quad f(x) = \arctg((\arctg x + x) \cdot \sqrt{x}), f'(x) = \frac{1}{1 + ((\arctg x + x) \cdot \sqrt{x})^2} \cdot \left(\left(\frac{1}{1+x^2} + 1 \right) \cdot \sqrt{x} + (\arctg x + x) \cdot \frac{1}{2\sqrt{x}} \right)$$

$$l) \quad f(x) = (\sin x - x^{-5})^2$$

$$m) \quad f(x) = \arctg(e^{(\ln x)^2})$$

$$n) \quad f(x) = (\cos(\sin(x^4)))^3$$

$$o) \quad f(x) = \cos(e^{6+\sqrt{x}})$$

$$p) \quad f(x) = \left(\frac{x^{-3}}{\ln x} + \frac{x^{-2}}{e^x} \right)^3$$

$$q) \quad f(x) = \ln \left(\frac{\sin(x^{-4})}{\cos(x^3)} \right)$$

$$r) \quad f(x) = \frac{e^{\arctg(x^{-1})}}{\sin(\ln x) - 8}$$

$$s) \quad f(x) = \ln(\ln(\arctg(\sqrt{x})))$$

$$t) \quad f(x) = 4 \cdot (5 + \arctg(e^x))$$

$$u) \quad f(x) = \arctg((\sin(x^{-3}))^6)$$

$$v) \quad f(x) = (\ln x - \sqrt{x}) \cdot x^{-5} \cdot e^x \quad f'(x) = \left(\frac{1}{x} - \frac{1}{2\sqrt{x}} \right) \cdot x^{-5} \cdot e^x + (\ln x - \sqrt{x}) \cdot ((-5x^{-6}) \cdot e^x + x^{-5} \cdot e^x)$$

$$w) \quad f(x) = \frac{\cos x}{\arctg x} \cdot \sqrt{\ln(6)}$$

ŘEŠENÍ

$$f'(x) = \frac{-3}{\left(\frac{x^5+x}{\sin x + \arctg x}\right)^4} \cdot \frac{(5x^4+1) \cdot (\sin x + \arctg x) - (x^5+x) \cdot \left(\cos x + \frac{1}{1+x^2}\right)}{(\sin x + \arctg x)^2}$$

$$f'(x) = \frac{1}{\sqrt{\cos x \cdot \sqrt{x}}} \cdot \frac{1}{2 \cdot \sqrt{\cos x \cdot \sqrt{x}}} \cdot \left((-\sin x) \cdot \sqrt{x} + \cos x \cdot \frac{1}{2\sqrt{x}} \right)$$

$$f'(x) = \cos(e^{e^{x^6}}) \cdot e^{e^{x^6}} \cdot e^{x^6} \cdot 6x^5$$

$$f'(x) = \frac{1}{x} + e^x - \frac{e^x \cdot x^3 - e^x \cdot 3x^2}{(x^3)^2}$$

$$f'(x) = (-\sin((\sin x)^3)) \cdot 3(\sin x)^2 \cdot \cos x$$

$$f'(x) = \frac{-\frac{3}{x^4} \cdot \ln x - \frac{1}{x^3} \cdot \frac{1}{x}}{(\ln x)^2} \cdot (\sin x - \cos x) + \frac{1}{\ln x} \cdot (\cos x - (-\sin x))$$

$$f'(x) = \frac{1}{x \cdot \arctg x} \cdot \left(\arctg x + x \cdot \frac{1}{1+x^2} \right)$$

$$f'(x) = \frac{(-(-3x^{-4})) \cdot \arctg x \cdot x^4 - (3-x^{-3}) \cdot \left(\frac{1}{1+x^2} \cdot x^4 + \arctg x \cdot 4x^3 \right)}{(\arctg x \cdot x^4)^2}$$

$$f'(x) = \frac{\left(e^x + \frac{1}{1+x^2} - \frac{1}{\ln x} \cdot \frac{1}{x} \right) \cdot 2 \cdot \sqrt{x} - (e^x + \arctg x - \ln(\ln x)) \cdot \left(2 \cdot \frac{1}{2\sqrt{x}} \right)}{(2 \cdot \sqrt{x})^2}$$

$$f'(x) = 2(\sin x - x^{-5}) \cdot (\cos x - (-5x^{-6}))$$

$$f'(x) = \frac{1}{1 + (e^{(\ln x)^2})^2} \cdot e^{(\ln x)^2} \cdot 2(\ln x) \cdot \frac{1}{x}$$

$$f'(x) = 3(\cos(\sin(x^4)))^2 \cdot (-\sin(\sin(x^4))) \cdot \cos(x^4) \cdot 4x^3$$

$$f'(x) = (-\sin(e^{6+\sqrt{x}})) \cdot e^{6+\sqrt{x}} \cdot \left(\frac{1}{2\sqrt{x}} \right)$$

$$f'(x) = 3 \left(\frac{x^{-3}}{\ln x} + \frac{x^{-2}}{e^x} \right)^2 \cdot \left(\frac{(-3x^{-4}) \cdot \ln x - x^{-3} \cdot \frac{1}{x}}{(\ln x)^2} + \frac{(-2x^{-3}) \cdot e^x - x^{-2} \cdot e^x}{(e^x)^2} \right)$$

$$f'(x) = \frac{1}{\frac{\sin(x^{-4})}{\cos(x^3)}} \cdot \frac{\cos(x^{-4}) \cdot (-4x^{-5}) \cdot \cos(x^3) - \sin(x^{-4}) \cdot (-\sin(x^3)) \cdot 3x^2}{(\cos(x^3))^2}$$

$$f'(x) = \frac{e^{\arctg(x^{-1})} \cdot \frac{1}{1+(x^{-1})^2} \cdot (-x^{-2}) \cdot (\sin(\ln x) - 8) - e^{\arctg(x^{-1})} \cdot (\cos(\ln x) \cdot \frac{1}{x})}{(\sin(\ln x) - 8)^2}$$

$$f'(x) = \frac{1}{\ln(\arctg(\sqrt{x}))} \cdot \frac{1}{\arctg(\sqrt{x})} \cdot \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$$

$$f'(x) = 4 \cdot \left(\frac{1}{1+(e^x)^2} \cdot e^x \right)$$

$$f'(x) = \frac{1}{1 + ((\sin(x^{-3}))^6)^2} \cdot 6(\sin(x^{-3}))^5 \cdot \cos(x^{-3}) \cdot (-3x^{-4})$$

$$f'(x) = \frac{(-\sin x) \cdot \arctg x - \cos x \cdot \frac{1}{1+x^2}}{(\arctg x)^2} \cdot \sqrt{\ln(6)}$$